Math 351: Questions for class discussion, 26th October

Brief look at Power series

Functions of one variable: continuity

**Review:**

**Define** conditional convergence; absolute convergence; alternating series; rearrangement

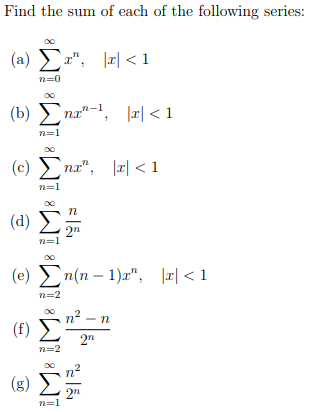
**State and prove** Cauchy’s test for conditional convergence of an alternating series.

**Prove** that absolute convergence implies convergence. Is the converse true?

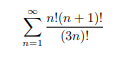
**State and prove** the rearrangement theorems.

**I Define:** *radius of convergence*

1. Prove the theorem: For each power series there is a unique *R* ≥ 0 such that converges absolutely for |x| < R and diverges for |x| > R.
2. Find the radius of convergence (and the sum) of each of the following power series:



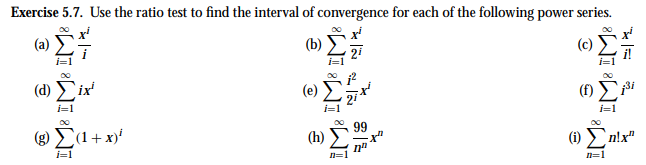
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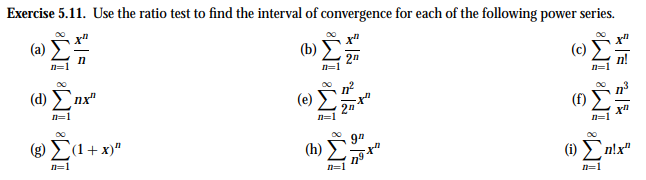


(i)



(Smith College exercises)





**II** Continuity & limits

1. Define *function, domain, graph.*

Let f(x) be defined on (a, b) and let p Define: **f(x) is continuous at x = p.**

Give both the “Mattuck” definition and the

Define: f(x) is **continuous** on (a, b).

1. Prove, using only the definition of continuity, that each of the following functions is continuous on the given interval.
2. f(x) = x2 on (-∞, ∞)
3. f(x) = 1/x on (0, ∞)
4. f(x) =  on (-∞, ∞)
5. f(x) = on (-∞, ∞)
6. Prove that g(x) = sin x is continuous on (-∞, ∞). Hint: show that |sin a – sin b| ≤ |a – b| .
7. Using (3) prove that
8. What are the four types of discontinuities?
9. Define: right-continuity, left-continuity. Define: **f(x) is continuous on [a, b].**
10. Using (3) prove that Which fact(s) about the Riemann integral are you taking for granted?
11. What are the four types of *discontinuities?* For each of the four, provide an example.
12. Let f be defined for x near *p*. Define: **the limit of f(x) as equals L.**
13. What is the relationship between continuity and limit? Define: limit as x
14. Prove each of the following results, using only the definition of limit.

