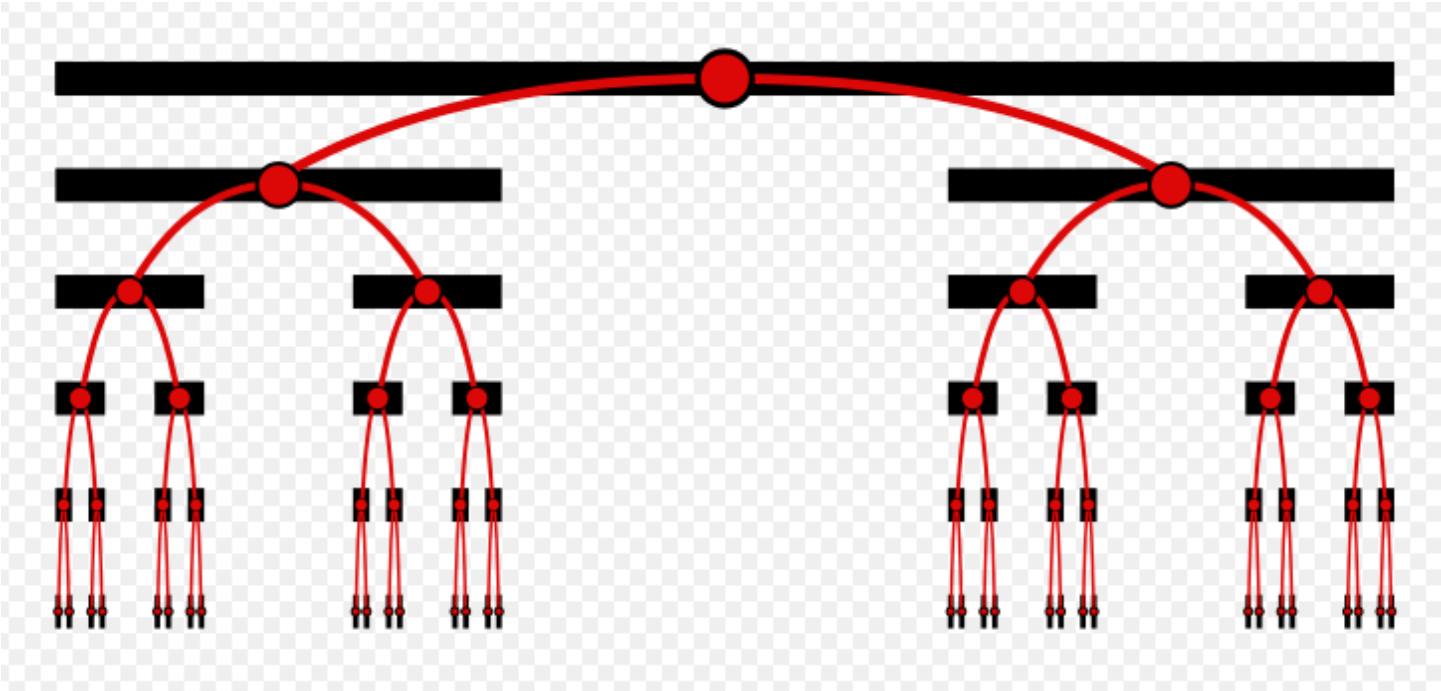
MATH 351: QUESTIONS FOR CLASS DISCUSSION, 29TH OCTOBER

FUNCTIONS OF ONE VARIABLE: CONTINUITY; LIMITS

Review:

Cantor ternary set



CONTINUITY & LIMITS

1. Define *function*, *domain*, *graph.*

Let f(x) be defined on (a, b) and let p ∈ (a, b). Define: **f(x) is continuous at x = p.**

Give both the "Mattuck" and the traditional ϵ, δ *definitions.*

2. Define: f(x) is **continuous** on (a, b).

3. Prove, using only the definition of continuity, that each of the following functions is continuous on the given interval.

(a) f(x) = x^2 on (-∞, ∞)

(b) f(x) = 1/x on (0, ∞)

(c) f(x) = $2x^3 - 4x$ on (-∞, ∞)

(d) f(x) = $\frac{x}{3+x^2}$ on (-∞, ∞)

4. Prove that g(x) = sin x is continuous on (-∞, ∞). Hint: show that |sin a – sin b| ≤ |a – b| .

5. What are the four types of discontinuities?

6. Define: right-continuity, left-continuity. Define: **f(x) is continuous on [a, b].**

7. Using (4) prove that $G(x) = \int_0^{\pi} \frac{\sin xt}{t}\, dt$ *is continuous everywhere.* Which fact(s) about the Riemann integral are you taking for granted?

8. Let f be defined for x near *p*. Define: **the limit of f(x) as $x \to p$ exists and equals L.**

9. (a) Let f(x) = 3x + 1. Prove, using only the definition of limit, that

$$\lim_{x\to 2} f(x) = 7$$

(b) Let g(x) = x^2. Prove, using only the definition of limit, that

$$\lim_{x\to 2} g(x) = 4$$

(c) Prove that

$$\lim_{x\to 2} x^2 + x - 1 = 5$$

(d) Prove that

$$\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$$

10. [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

True or False? Justify!

(a) If a particular δ has been constructed as a suitable response to a particular ε challenge, then any smaller positive δ will also suffice.

(b) If $\lim_{x\to b} f(x) = L$ and b happens to be in the domaing of f, then L = f(b).

(c) If $\lim_{x\to b} f(x) = L, then \lim_{x\to b} 3\,(f(x) - 2)^2 = 3(L-2)^2$

(d) If $\lim_{x\to b} f(x) = 0, then \lim_{x\to b} f(x)g(x) = 0,$

for any function g ($with\ domain\ equal\ to\ the\ domain\ of\ f$).

11. What is the relationship between continuity and limit? Define: limit as x → ∞ *or* x → −∞.

12. Prove each of the following results, using only the definition of limit.

(a) $\lim_{x\to 0} x \sin\frac{1}{x}$

(b) $\lim_{x\to 3+} \frac{|x^2-9|}{x-3}$

(c) $\lim_{x\to\infty} \frac{1}{3+x^2}$

(d) $\lim_{x\to 1} \frac{x^3-125}{x-5}$