Math 351: class discussion, 3 October 2018



1. Complete the proof of the Nested Intervals Theorem.
2. Show that, for any $α\in R, there exists a sequence of nested intervals having intersection \left\{α\right\}.$
3. Example. Let $a\_{0}=0, and for n\geq 1, let a\_{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+…+(-1)^{n-1}\frac{1}{n}$

Using the nested interval theorem, prove that $\left\{a\_{n}\right\} converges.$

1. Definition: K is a ***cluster point*** of $\left\{a\_{n}\right\} $means that, for all  > 0, there exist infinitely many *n* for which

|an – K| < 

1. Find any (and all) cluster points for each of the following sequences:

(a) ½, 1/3, ¼, 1/5, …

(b) 1, 2, 3, 4, 5, …

(c) 1, 0, 1, 0, 1, 0, …

(d) 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, …

(e) 2, 3, 5, 7, 11, 13, …

1. Prove the Cluster Point Theorem: A sequence $\left\{a\_{n}\right\}$ has a cluster point, K, if and only if there exists a subsequence of $\left\{a\_{n}\right\}$ that converges to K.
2. Prove the Bolzano-Weierstrass Theorem: A bounded sequence $\left\{a\_{n}\right\} has a convegent subsequence.$

Exercises:

1. Give an example of each of the following or argue that no such example exists:
2. A sequence that has a subsequence that is bounded but contains no subsequence that converges.
3. A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these two values.
4. A sequence that contains subsequences converging to every point in the infinite set

{1/2, 1/3, ¼, 1/5, …}.

1. A sequence that contains subsequences converging to every point in the infinite set

{1, 1/2, 1/3, ¼, 1/5, …}, and no subsequences converging to points outside of this set.

1. Decide whether each of the following statements is True or False. Provide either a brief justification or a counterexample.
2. If every proper subsequence of $\left\{x\_{n}\right\}$ converges, then $\left\{x\_{n}\right\}$ converges as well.
3. If $\left\{a\_{n}\right\} contains a divergent subsequence, then \left\{a\_{n}\right\} diverges.$
4. If $\left\{a\_{n}\right\} is bounded and diverges, then there exist two subsequences of \left\{a\_{n}\right\} that converge$

to different limits.

1. If $\left\{a\_{n}\right\} is monotone and contains a convergent subsequence, then \left\{a\_{n}\right\} converges.$



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