MATH 351: QUESTIONS FOR CLASS DISCUSSION, 31TH OCTOBER

CONTINUITY; LIMITS

REVIEW

1. Prove, using only the definition of continuity, that each of the following functions is continuous on the given interval.
   (a) $f(x) = x^3$ on $(-\infty, \infty)$
   (b) $f(x) = 1/x$ on $(0, \infty)$
   (c) $f(x) = x^3 - 4x$ on $(-\infty, \infty)$
   (d) $f(x) = \frac{x^2}{1+x^2}$ on $(-\infty, \infty)$
2. Prove that $g(x) = \sin x$ is continuous on $(-\infty, \infty)$. Hint: show that $|\sin a - \sin b| \le |a - b|$.
3. What are the four types of discontinuities?
4. Define: right-continuity, left-continuity. Define: **f(x) is continuous on [a, b].**
5. Using (2) prove that $G(x) = \int_0^{\pi} \frac{\sin xt}{t}\, dt$ *is continuous everywhere*. Which fact(s) about the Riemann integral are you taking for granted?

LIMITS

6. Let f be defined for x near p. Define: **the limit of f(x) as $x \to p$ exists and equals L.**
   Define right-hand limit and left-hand limit.
7. (a) Let $f(x) = 3x + 1$. Prove, using only the definition of limit, that

$$\lim_{x \to 2} f(x) = 7$$

   (b) Let $g(x) = x^2$. Prove, using only the definition of limit, that

$$\lim_{x \to 2} g(x) = 4$$

   (c) Prove that

$$\lim_{x\to 2}\ (x^2 + x - 1) = 5$$

(d) Prove that

$$\lim_{x\to 4} \frac{1}{x} = \frac{1}{4}$$

8. [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

True or False? Justify!

(a) If a particular δ has been constructed as a suitable response to a particular ε challenge, then any smaller positive δ will also suffice.

(b) If $\lim_{x\to b} f(x) = L$ and b happens to be in the domain of f, then L = f(b).

(c) If $\lim_{x\to b} f(x) = L, then \lim_{x\to b} 3\,(f(x) - 2)^2 = 3(L-2)^2$

(d) If $\lim_{x\to b} f(x) = 0, then \quad \lim_{x\to b} f(x)g(x) = 0,$

for any function g $(with\ domain\ equal\ to\ the\ domain\ of\ f)$.

9. What is the relationship between continuity and limit? Define: limit as x $\to \infty$ *or* x $\to -\infty$.

10. Prove each of the following results, *using only the definition* of limit.

(a) $\lim_{x\to 0} x\sin\frac{1}{x}$

(b) $\lim_{x\to 3+} \frac{|x^2-9|}{x-3}$

(c) $\lim_{x\to\infty} \frac{1}{3+x^2}$

(d) $\lim_{x\to 1} \frac{x^3-125}{x-5}$

11. Define infinite limits.

12. Explain the relationship between continuity and limits.

13. (Mattuck) Does "absolute continuity" imply continuity? That is, if |f(x)| is continuous on I, will f(x) be continuous on I?

14. State the Algebraic Limit Theorems for functions.

15. [S. Abbott, **Understanding Analysis**, 2nd edition, Springer (2016)]

Provide an example of each or explain why the request is impossible.

(a) Two functions f and g, neither of which is continuous at 0 but such that f(x)g(x) and f(x) + g(x) are continuous at 0.

(b) A function f(x) continuous at 0 and g(x) not continuous at 0 such that f(x) + g(x) is continuous at 0.

(c) A function f(x) continuous at 0 and g(x) not continuous at 0 such that f(x)g(x) is continuous at 0.

(d) A function f(x) not continuous at 0 such that $f(x) + \frac{1}{f(x)}$ is continuous at 0.

(e) A function f(x) not continuous at 0 such that is continuous at 0.

16. State and prove the Squeeze Theorem for limits.

17. State the Limit Location Theorem for functions.

18. State the Function Location Theorem.

19. What is Sequential Continuity?

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