Math 351: class discussion, 5 October 2018

1. Complete the proof of the Bolzano-Weierstrass Theorem:

A *bounded* sequence

1. Define Cauchy sequence.
2. How are Cauchy sequences related to convergent sequences? What is meant by “Cauchy criterion for convergence”?
3. Show directly that

*Hint:* First note that

1. Consider the following statement: “Given for n >>1”

Give an *example* of an increasing sequence that satisfies this condition, yet is not a Cauchy sequence.

1. Given a sequence for all *n* and *k*. Prove that is Cauchy.
2. Given a sequence for all *n*. Must it follow that is Cauchy?

**Exercises:** (S. Abbott, **Understanding Analysis**, 2nd edition, Springer)

1. Give an example of each of the following or argue that no such example exists:
2. A sequence that has a subsequence that is bounded but contains no subsequence that converges.
3. A sequence that does not contain 0 or 1 as a term but contains subsequences converging to each of these two values.
4. A sequence that contains subsequences converging to every point in the infinite set

{1/2, 1/3, ¼, 1/5, …}.

1. A sequence that contains subsequences converging to every point in the infinite set

{1, 1/2, 1/3, ¼, 1/5, …}, and no subsequences converging to points outside of this set.

1. Decide whether each of the following statements is True or False. Provide either a brief justification or a counterexample.
2. If every proper subsequence of converges, then converges as well.
3. If
4. If

to different limits.

1. If
2. Give an example of each of the following or argue that no such example exists:
3. A Cauchy sequence that is not monotone.
4. A Cauchy sequence with an unbounded subsequence.
5. A divergent monotone sequence with a Cauchy subsequence.
6. An unbounded sequence containing a subsequence that is Cauchy.
7. If are Cauchy sequences, then one easy way to prove that is Cauchy is to use the Cauchy criterion. Explain!
8. Give a direct argument that is Cauchy that does not use the Cauchy criterion.
9. Do the same for the product, .
10. Let be Cauchy sequences. Decide whether or not each of the following is Cauchy, justifying each conclusion.

13. where refers to the greatest integer less than or equal to x.

