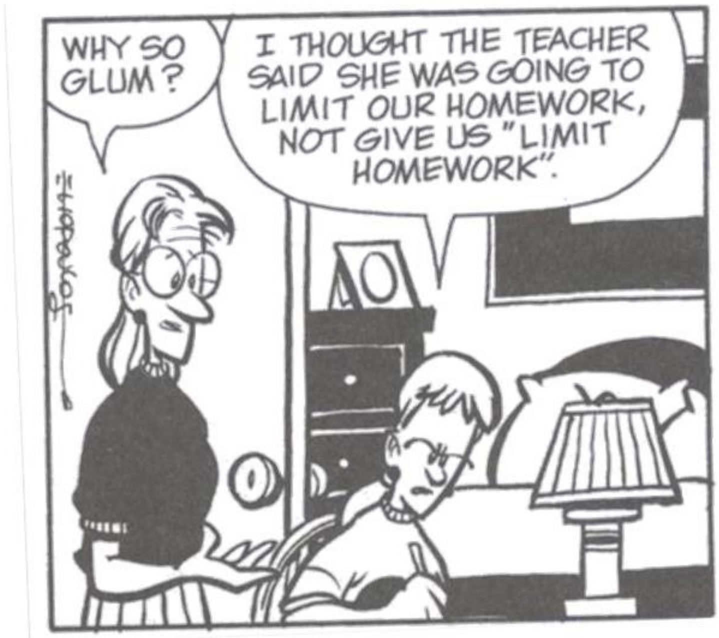
MATH 351: QUESTIONS FOR DISCUSSION, 10 SEPTEMBER 2018

Introduction to Limits



❖ EXERCISES *(continued from last meeting)*

1. Define *absolute value*. State and prove the multiplicative property and the triangle inequality. State and prove, using induction, the extended triangle inequality. Show that the inequality $|a - b| \geq |a| - |b|$ (called the *difference form* of the triangle inequality) follows directly from the triangle inequality.
2. Let $s_n = c_1 \cos t + c_2 \cos 2t + \ldots + c_n \cos nt$ where $c_j = \frac{1}{2^j}$. Prove that $\{s_n\}$ is bounded by 1.
3. Let $b_n = \frac{2n+7}{n-9}$. Guess the limit, L, of the sequence $\{b_n\}$. For which (positive) values of n is $b_n \underset{0.1}{\approx} L$?
4. Let $b_n = \frac{n+3}{n^2+n+19}$. Guess the limit, L, of the sequence $\{b_n\}$. For which (positive) values of n is $b_n \underset{0.001}{\approx} L$?
5. Let $\{b_n\}$ be a sequence. Give the definition of: $\{b_n\}$ **converges**.

   Define: $\{b_n\}$ **converges to L**.
6. Let $z_n = \frac{1789 + \cos(2018n)}{3n+88}$. Prove that $\{z_n\}$ converges to a limit L.
7. Let $e_n = \sqrt{n^2 + 5n + 1789} - \sqrt{n^2 + n + 1689}$. Prove that $\{e_n\}$ converges to a limit L.

❖ MORE ON LIMITS

1. Prove that if a sequence converges to L, then L is unique.
2. Prove that if $\{a_n\}$ is a non-negative sequence converging to 0, the sequence $\{\sqrt{a_n}\}$ must converge to 0 as well.
3. Define $lim\ a_n = \infty$.
4. Prove that the sequence $a_n = 1 + n^2 \to \infty$.

5. Which of the following sequences tend to ∞? For those that do, prove it.

(a) $(-1)^2$

(b) $\frac{n}{n+4}$

(c) $(-1)^n n^2$

(d) $\sqrt[3]{n+1}$

(e) $1+ n^2$

(f) $(-1)^n + \sin n + e^n$

(g) $\sin n + \ln n$

6. Prove that the sequence $a_n = \frac{1}{n+1} + \frac{3}{n+2}$ converges.

7. State the K-ε Principle.

8. Using the K-ε Principle prove that the sequence $a_n = \frac{1}{n+1} + \frac{3}{n+2}$ converges.

9. Using the K-ε Principle prove that the sequence $b_n = \frac{n}{2n+1} + \frac{n}{n+2}$ converges.

10. Prove that $\lim_{n\to\infty} \frac{15n^3+2018}{5n^3+n+1}$ *exists.*

11. Prove that $\ln(\ln n) \to \infty$.

12. Prove that $\frac{e^{3n}}{2018+e^n} \to \infty$.

13. Prove the theorem: $If\ a > I\ then \lim_{n\to\infty} a^n = \infty$.

$Hint\colon\ let\ a = 1 + k\ where\ k > 0.\ Then\ apply\ the\ binomial\ theorem.$

14. *Prove the Corollary to the Theorem above, viz.*

If $0 < a < I$ then $\lim_{n\to\infty} a^n = 0$.

15. Find $\lim_{n\to\infty} \left(1 - \frac{\alpha}{100000}\right)^n$ given that $100000 > \alpha > 0$.

16. Find $\lim_{n\to\infty} \left(\sin^8 \frac{\pi}{5}\right)^n$.

17. Prove that every convergent sequence is bounded.

Everything should be made as simple as possible, but not simpler.

– Albert Einstein, **Reader's Digest** (Oct. 1977)