MATH 351: QUESTIONS FOR CLASS DISCUSSION, 12 SEPTEMBER 2018

We shouldn't hunt rabbits Dad, we should breed them!
According to Fibonacci,
we would then have an endless food supply...

1. *Review:*

(a) Prove that $\lim_{n\to\infty} \frac{20n+9}{10n-3} = 2$.

(b) Determine whether $\lim_{n\to\infty} \left(\sqrt{n+2018} - \sqrt{n}\right)$ exists. If so, find its limit and verify using the definition of limit.

(c) Determine whether $\lim \frac{n^2+1}{(n+1)^2}$ exists. If so, find its limit and verify using the definition of limit.

(d) Determine whether $\lim_{n\to\infty} \frac{n^2+13}{\sqrt{n^2+3n}}$ exists. If so, find its limit and verify using the definition of limit.

2. Prove that if a sequence converges to L, then L is unique.

3. Prove that every convergent sequence is bounded.

4. Prove that if $\{a_n\}$ is a non-negative sequence converging to 0, the sequence $\{\sqrt{a_n}\}$ must converge to 0 as well.

5. Define $lim\ a_n = \infty$.

6. Prove that the sequence $a_n = 1 + n^2 \to \infty$.

7. Which of the following sequences tend to ∞? For those that do, prove it.

(a) $(-1)^2$

(b) $\frac{n}{n+4}$

(c) $(-1)^n n^2$

(d) $\sqrt[3]{n+1}$

(e) $1+n^2$

(f) $(-1)^n + \sin n + e^n$

(g) $\sin n + \ln n$

8. State the K-ε Principle. Prove that the sequence $a_n = \frac{1}{n+1} + \frac{3}{n+2}$ converges.

9. Using the K-ε Principle prove that the sequence $b_n = \frac{n}{2n+1} + \frac{n}{n+2}$ converges.

10. Prove that $\ln(\ln n) \to \infty$.

11. Prove that $\frac{e^{3n}}{2018+e^n} \to \infty$.

12. Prove the theorem: $If\ a > I\ then\ \lim_{n\to\infty} a^n = \infty$.

$Hint$: $let\ a = 1 + k\ where\ k > 0.\ Then\ apply\ the\ binomial\ theorem.$

13. $Prove\ the\ Corollary\ to\ the\ Theorem\ above, viz.$

$$\text{If } 0 < a < I \text{ then } \lim_{n\to\infty} a^n = 0.$$

14. Prove that every convergent sequence is bounded.

15. (a) Find $\lim_{n\to\infty} \left(1 - \frac{\alpha}{100000}\right)^n$ given that $\alpha > 0$.

(b) Find $\lim_{n\to\infty} \left(\cos^8\left(\frac{\pi}{5}\right)\right)^n$

(c) Find $\lim_{n\to\infty} (\ln 3)^n$

16. Let $c_n = \int_0^1 (x^2+3)^n dx$. Find $\lim_{n\to\infty} c_n.$

17. Let $d_n = \int_0^1 (x^2+1)^n dx$. Find $\lim_{n\to\infty} d_n.$

18. Let $s_n = \int_0^{\pi/2} sin^n\ x\ dx$. Find $\lim_{n\to\infty} s_n.$

19. Prove that if $a_n \to L\ and\ b_n \to M\ then\ a_n + b_n \to L + M$.