MATH 351: CLASS DISCUSSION, 17 SEPTEMBER

LIMIT THEOREMS; SUBSEQUENCES

One of the many "find *N* given ε" demons

1. Using the error-form principle, prove that if $a_n \to L, then\ |a_n| \to |L|$. Is the converse true? Proof or counterexample.
2. $If\ a_n = \frac{25n^3+n^2-n+2018}{5n^3-13n^2+1789} = 5$ find an M such that $|e_n| < 0.02$ for all n > M.
3. *Using the limit theorems prove that*

$$\lim_{n\to\infty} \frac{25n^3+n^2-n+2018}{5n^3-13n^2+1789} = 5$$

4. *Review:* Let |a| < 1. For n ≥ 1, let $S_n = 1 + a + a^2 + \cdots + a^n$.

   Using the error-form principle, prove that $\lim_{n\to\infty} S_n = \frac{1}{1-a}$.

   Hint: Show that $e_n = -\frac{a^{n+1}}{1-a}$.

5. State the three main limit theorems for sequences and prove each using the error-form principle.

6. *(Review of calculus)* Derive a recursive form of Newton's method for finding roots of a differentiable function y = f(x).
7. Applying Newton's method to the polynomial p(x) = $x^2 - 2$, find a recursive sequence that converges to $\sqrt{2}$. Use the error-form principle to prove the result.
8. Prove the **Squeeze theorem**, *viz:*

   Given three sequences, $\{a_n\},\ \{b_n\},\ \{c_n\}\ satisfying\ \ a_n \le b_n \le c_n\ for\ n \gg 1$.

   Suppose that $a_n \to L\ and\ c_n \to L\ .\ \ Then\ b_n \to L$.

9. *Example:* Let α be given. Then $\sqrt[n]{2+\cos n\alpha} \to 1$.

   Hint: Use the result of exercise 3.4/5, *viz.*

   If c > 0 then $\sqrt[n]{c} \to 1$.

10. *Example:* Prove that $\lim_{n\to\infty} \frac{\ln(n!)}{n \ln n} = 1$.

    Hint: $\ln(n!) = \ln 1 + \ln 2 + \ln 3 + \cdots + \ln n$

11. Find $\lim n(a + \cos n\,\pi)$ *for different values of a.*

12. Prove the **Sequence location theorem**, *viz:*

$If\ \ \{a_n\}\ converges$ and lim a_n < M, then a_n < M for n >> 1.

13. State the corresponding version of the Sequence location theorem for a convergent sequence bounded below.

14. Prove the **Limit location theorem**, *viz:*

$If\ \ \{a_n\}\ converges\ and\ \exists M\ such\ that\ a_n \leq M\ for\ n \gg 1,$ then $\lim_{n\to\infty} a_n\ \leq M.$

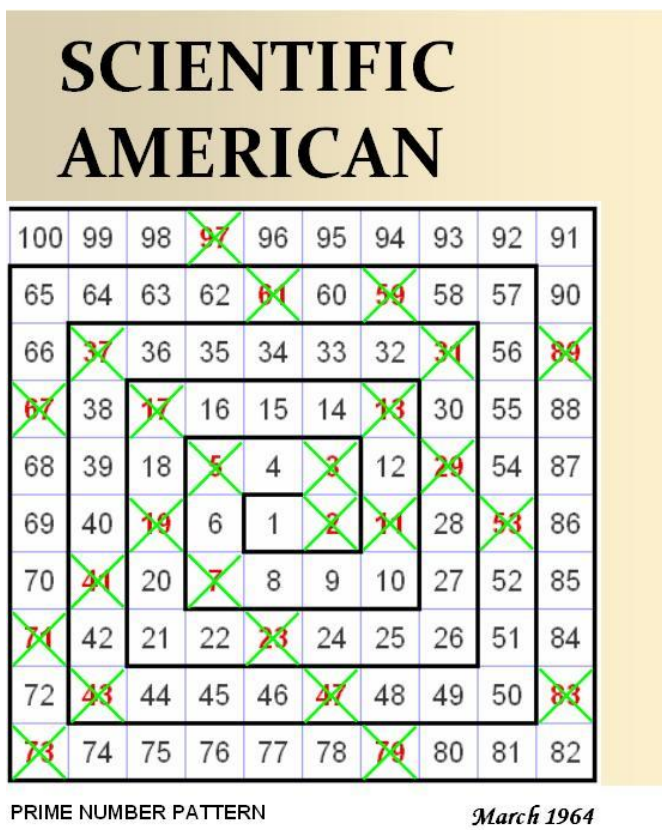
15. State the corresponding version of the Limit location theorem for a convergent sequence *bounded below*.

16. Prove the following useful Corollary to the LLT, *viz.*

Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences and assume that $a_n \leq b_n$ for n >> 1. Then lim a_n = lim b_n.

17. *Define* **Subsequence** of a sequence $\{a_n\}$.

18. State the **Subsequence theorem**.



Prime number pattern in the Ulam spiral