Math 351: class discussion, 19 September

Limit Theorems



*Review:*

1. Using the error-form principle, prove that if Is the converse true? Proof or counterexample.
2. find an M such that |en| < 0.02 for all n > M.
3. *Review:* Let |a| < 1. For n ≥ 1, let

Using the error-form principle, prove that *Hint:* Show that

1. State the three main limit theorems for sequences and prove each using the error-form principle.
2. *Using the limit theorems prove that*
3. *(Review of calculus)* Derive a recursive form of Newton’s method for finding roots of a differentiable function y = f(x).
4. (a) Applying Newton’s method to the polynomial p(x) = x2 – 2, find a recursive sequence that converges to Use the error-form principle to prove the result.

(b) *Practice exercise:* using (a) as a guide, find a recursive sequence that converges to

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*Two more major theorems of the week (in addition to the three limit theorems that you have stated in question 4 above.)*

1. Prove the **Squeeze theorem**, *viz:*

Given three sequences,

Suppose that

1. *Example:* Let  be given. Then

*Hint:* Use the result of exercise 3.4/5, *viz.* If c > 0 then

1. *Example:* Prove that *Hint:*
2. Find *) for different values of a.*
3. Prove the **Sequence location theorem**, *viz:*

 and lim an < M, then an < M for n >> 1.

1. State the corresponding version of the Sequence location theorem for a convergent sequence bounded below.
2. (a) Prove the **Limit location theorem**, *viz:*

then

State the corresponding version of the Limit location theorem for a convergent sequence *bounded below*.

1. Prove the following useful Corollary to the LLT, *viz.* Let {an} and {bn} be convergent sequences and assume that

an ≤ bn for n >> 1. Then lim an = lim bn.

**Exercises from Apostol**

In Exercises 1 through 10, compute the limits and explain which limit theorems you are using in each case.

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*Calculus has its limits.*

* Anonymous