Math 351: class discussion, 21 September

location Theorems; subsequences

*Review:*

1. Using the error-form principle, prove that if Is the converse true? Proof or counterexample.
2. *(Review of calculus)* Derive a recursive form of Newton’s method for finding roots of a differentiable function y = f(x).
3. (a) Applying Newton’s method to the polynomial p(x) = x2 – 5, find a recursive sequence that converges to Use the error-form principle to prove the result.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Three more useful theorems of the week (in addition to the three major limit theorems that we have recently studied).*

1. Prove the **Squeeze theorem**, *viz:*

Given three sequences,

Suppose that

1. Prove: If c > 0 then Hint: First consider the case: c > 1.

Then, for the case 0 < c < 1, consider.

1. *Example:* Let  be given. Then
2. *Example:* Prove that *Hint:* .

*Note: This is a simplified version of Sterling’s formula.*

1. Find *) for different values of a.*
2. Prove the **Sequence location theorem**, *viz:*

 and lim an < M, then an < M for n >> 1.

1. State the corresponding version of the Sequence location theorem for a convergent sequence bounded below.
2. (a) Prove the **Limit location theorem**, *viz:*

then

State the corresponding version of the Limit location theorem for a convergent sequence *bounded below*.

1. Prove the following useful Corollary to the LLT, *viz.* Let {an} and {bn} be convergent sequences and assume that

an ≤ bn for n >> 1. Then lim an ≤ lim bn.

1. Using the *Sequence location theorem*, give a more concise proof of the limit theorem for quotients, *viz.*

Given an n

*Hint:* Consider case 1: L > 0.

1. Define ***subsequence*** of a sequence {an}. Consider the sequence 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, …

Find several convergent subsequences and several divergent sequences.

1. (a) If {an} is monotone for n>>1, is every subsequence of {an} monotone for n>>1?

(b) If {an} is bounded for n>>1, is every subsequence of {an} bounded for n>>1?

(c) If an , must every subsequence bn

1. State and prove the **Subsequence theorem**.
2. Prove, using the subsequence theorem, that the sequence {} does not converge. Hint: Consider the region where .
3. Prove, using the subsequence theorem, that the sequence { does not converge.
4. What is meant by a subsequence of a sequence?
5. Prove the Subsequence Theorem: If {an} converges, then every subsequence also converges, and to the same limit.
6. Prove that Hint: Consider the region where .

