Math 351: class discussion, 24 September

Subsequences; nested intervals theorem



*Review:*

1. *Example:* Let  be given. Then
2. *Example:* Prove that *Hint:* .

*Note: This is a simplified version of Sterling’s formula.*

1. Prove the **Sequence Location Theorem**, *viz:*

 and lim an < M, then an < M for n >> 1.

1. State the corresponding version of the Sequence location theorem for a convergent sequence bounded below.
2. (a) Prove the **Limit Location Theorem**, *viz:*

then

 State the corresponding version of the Limit location theorem for a convergent sequence *bounded below*.

1. Prove the following useful Corollary to the LLT, *viz.* Let {an} and {bn} be convergent sequences and assume that

an ≤ bn for n >> 1. Then lim an ≤ lim bn.

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1. Define ***subsequence*** of a sequence {an}. Consider the sequence 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, …

Find several convergent subsequences and several divergent sequences.

1. (a) If {an} is monotone for n>>1, is every subsequence of {an} monotone for n>>1?

(b) If {an} is bounded for n>>1, is every subsequence of {an} bounded for n>>1?

(c) If an , must every subsequence bn

1. Find an example of a sequence that has countably many subsequences each having a distinct limit.
2. Find an example of a sequence that has uncountably many subsequences, each having a distinct limit.
3. State and prove the **Subsequence Theorem**.
4. Prove, using the subsequence theorem, that the sequence {} does not converge.
5. Prove, using the subsequence theorem, that the sequence { does not converge.

Hint: Consider the region where .

1. What is meant by a **nested sequence of intervals**? Give several examples, some of which have intersection (a) containing only one point; (b) containing infinitely many points; (c) empty.
2. State the **Nested Intervals Theorem**. Give a proof. Can any of the hypotheses be eliminated or weakened?
3. Show that, given any
4. Example. Let

Using the nested interval theorem, prove that

1. Define *cluster point* of a sequence.
2. Find any (and all) cluster points for each of the following sequences:

(a) ½, 1/3, ¼, 1/5, …

(b) 1, 2, 3, 4, 5, …

(c) 1, 0, 1, 0, 1, 0, …

(d) 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, …

(e) 2, 3, 5, 7, 11, 13, …