Math 351: class discussion, 26 September

nested intervals theorem



1. (a) If {an} is monotone for n>>1, is every subsequence of {an} monotone for n>>1?

(b) If {an} is bounded for n>>1, is every subsequence of {an} bounded for n>>1?

(c) If an , must every subsequence bn

1. Find an example of a sequence that has countably many subsequences each having a distinct limit.
2. State and prove the **Subsequence Theorem**.
3. Prove, using the subsequence theorem, that the sequence {} does not converge.
4. Prove, using the subsequence theorem, that the sequence { does not converge.

Hint: Consider the region where .

1. What is meant by a **nested sequence of intervals**? Give several examples, some of which have intersection (a) containing only one point; (b) containing infinitely many points; (c) empty.
2. State the **Nested Intervals Theorem**. Give a proof. Can any of the hypotheses be eliminated or weakened?
3. Show that, given any
4. Example. Let

Using the nested interval theorem, prove that

1. Definition: K is a ***cluster point*** of means that, for all  > 0, there exist infinitely many *n* for which

|an – K| < 

1. Find any (and all) cluster points for each of the following sequences:

(a) ½, 1/3, ¼, 1/5, …

(b) 1, 2, 3, 4, 5, …

(c) 1, 0, 1, 0, 1, 0, …

(d) 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, …

(e) 2, 3, 5, 7, 11, 13, …