Math 351: class discussion, 26 September

nested intervals theorem



1. (a) If {an} is monotone for n>>1, is every subsequence of {an} monotone for n>>1?

 (b) If {an} is bounded for n>>1, is every subsequence of {an} bounded for n>>1?

 (c) If an $\rightarrow \infty $, must every subsequence bn $\rightarrow \infty ?$

1. Find an example of a sequence that has countably many subsequences each having a distinct limit.
2. State and prove the **Subsequence Theorem**.
3. Prove, using the subsequence theorem, that the sequence {$\sin((\frac{nπ}{2}))$} does not converge.
4. Prove, using the subsequence theorem, that the sequence {$\sin(n\})$ does not converge.

Hint: Consider the region where $\sin(n) \geq \frac{\sqrt{2}}{2}$.

1. What is meant by a **nested sequence of intervals**? Give several examples, some of which have intersection (a) containing only one point; (b) containing infinitely many points; (c) empty.
2. State the **Nested Intervals Theorem**. Give a proof. Can any of the hypotheses be eliminated or weakened?
3. Show that, given any $α\in R, there exists a sequence of nested intervals having intersection \left\{α\right\}.$
4. Example. Let $a\_{0}=0, and for n\geq 1, let a\_{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+…+(-1)^{n-1}\frac{1}{n}$

Using the nested interval theorem, prove that $\left\{a\_{n}\right\} converges.$

1. Definition: K is a ***cluster point*** of $\left\{a\_{n}\right\} $means that, for all  > 0, there exist infinitely many *n* for which

|an – K| < 

1. Find any (and all) cluster points for each of the following sequences:

(a) ½, 1/3, ¼, 1/5, …

(b) 1, 2, 3, 4, 5, …

(c) 1, 0, 1, 0, 1, 0, …

(d) 1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, …

(e) 2, 3, 5, 7, 11, 13, …