Math 351 solutions: HW II

*Solutions to the following:*

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**Exercise 2.6.3**

Prove that if

**Solution:** Choose.

Then, for n > N,

**Exercise 2.6.4 (b)**

Prove that {an} is decreasing for n>>1, if **a6** = 1 and

(b)

**Solution:**

**Problem 2.1**

Let {an} be a sequence. We construct from it another sequence {bn} as follows:



1. Prove that if {an} is increasing, then {bn} is also increasing.
2. Prove that if {an} is bounded above, then {bn} is also bounded above.

 **Solution:**

***Part (a):***Assume that {an} is increasing. Then an+1 ≥ an for all n∈**N**.

Now



Hence{bn} is increasing.

***Part (b):***Assume that {an} is bounded above. Then, by definition, there exists *c*∈**R** such that an ≤ c for all

n ≥ 1. Thus



and hence, for all n ≥ 1:



Thus {bn} is bounded above.

**Exercise 3.1.1 (a)**

Show that directly from the definition of limit.

**Solution**: Let

Then, invoking the triangle inequality,

**Exercise 3.2.3 (a)**

Let Prove that an → 0.

1. Prove that {an} converges.

**Solution:**

***Part (a):*** *The sequence {an} is bounded above by 1 since:*

*Next, observe that:*



*Thus {an} is strictly increasing.*

*Now, since {an} is bounded above and increasing, we invoke the Completeness Theorem to conclude that {an} converges.*

**Problem 3.1:**

Let {an} be a sequence. As before, let {bn} be defined as follows:



1. Prove that if an → 0, then bn → 0.
2. Deduce from part (a) in a few lines that if an → L, then bn → L.

**Solution:**

*Part (a):* Assume that an → 0.

Let ε > 0 be given. Choose n\*∈ **N,** n ≥ 2, such that |an – 0| < ε when n ≥ n\*.

Choose m\*∈ **N** such



Next, let r\* = max{n\*, m\*}.

Thus, invoking the K-principle, bn 0.

 *Part (b):* Assume that an → L.

Define dn = an – L. We first show that dn → 0. Let  > 0 be given. Then an is -close to *L* for large *n*. Using the additivity property, dn = an – Lis -close to *L* – *L = 0*. So dn → 0.

Using part (a), we obtain yn 0, where



Since



 we obtain



Using the “-close”argument given above, we obtain:



**Problem 3.2:** To prove an was large if a > 1, we used “Bernoulli’s inequality.”



**Solution:** Assume that h0 and that

Let P(n) be the statement that (1 + h)n

Base case: n = 0

LHS =

RHS = 1 + 0 h = 1

This establishes the base case.

Inductive step: Assume that

 Then

This proves P(n+1) which completes induction.



**Solution:**

If , then the induction argument above remains valid.

The key observation is that, And so then the inequality

 *is still valid because .*