Math 351 solutions: HW III

*Solutions to the following:*

Mattuck **Submit:**

pg 47 / exercise 3.6.1 (a)

pg 48 / exercise 3.7.1

pg 73 / exercise 5.1.4

pg 74 / exercise 5.2.4

pg 75 / problem 5.1(a, b)

**Exercise 3.6.1**

Prove the following without attempting to evaluate the limt explicitly.

**Solution:** Observe that, on the interval [1, 2], ln x is non-negative. Also note that ln 2 ≤ ln e = 1.

Since y = ln x is an *increasing* function,

Recall that, for 0 < a < 1,

Next, let

Using properties of the Riemann integral:

**Exercise 3.7.1: Show that the sequence {an}, defined below, converges to 0.**

****

***Solution:*** *Let  > 0 be given. Let \* = min{, ½}. Define fn(x) = (1 – x2)n for 0 ≤ x ≤ 1, and let b = f(\*).*

*Notice that fn is a decreasing non-negative function on [0, 1] with maximum value of 1 at x = 0.*

*Since 0 < b < 1, it follows from Theorem 3.4 that bn → 0.*

*Choose M such that | bn – 0| < \* when n ≥ M. Now, using basic properties of the Riemann integral, for n > M:*

**

*Thus, invoking the K-principle, we obtain the desired result: an → 0.*

**Exercise 5.1.4**

Given that an/bn →L, bn ≠ 0 for all n∈**N**, and bn→0, prove that an→0.

***Proof:***

*Invoking the Product Rule for limits we know that the product of two convergent sequences converges: Thus an = (an /bn) (bn) converges and its limit is the product of the limits of the two convergent sequences:*

*lim an = lim (an /bn) lim bn = (L) (0) = 0.*

**Exercise 5.2.4**

**

Prove that {an} converges and find its limit.

***Proof:***

*We conjecture that lim an = ln 2. To prove this we compare area under the curve y = 1/x from x = n+1 to x = 2n+1 with upper rectangles of base width 1. This area is smaller than an. Hence*

**

*Similarly, we compare the area under the curve y = 1/x from x = n + 1 to x = 2n+1 with lower rectangles of base width 1. This area is smaller than an. Thus*



*Finally:*

**

*Since, using the laws of limits, (2n+1)/(n+1) = (2+(1/n))/(1+(1/n)) →2, ln((2n+1)/n )→ln 2 and since*

*(2n)/(n–1) →2, ln((2n)/(n–1))→ln 2. Invoking the Squeeze Theorem, we obtain: an → ln 2.*

**Problem 5.1 (a) If an ≥ 0 for all n∈N and an → L, then (an)1/2 → L1/2.**

Criticize the “proof” given.

*Solution: This “proof” assumes that exists. This is a result which must be proven!*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Problem 5.1 (b) If an ≥ 0 for all n∈N and an → L, then (an)1/2 → L1/2.**

***Solution:*** *Note that the Limit Location Theorem implies that L ≥ 0; so L1/2 is real.*

***Case I:*** *L ≠ 0*

*Let en = (an)1/2 – L1/2*

*Let  > 0 be given. Then*

**

***Case II:*** *L = 0*

*Let > 0 be given. Then, since an → 0, |an – 0| <  2  for n >>1.*

*Hence (an)1/2 <  for n >>1.*

*Thus (an)1/2 → 0.*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_