**MATH 351 Practice TEST II**



**Part A**

Definitions and statements of theorems.

**Part B**  *Each of the following assertions is false.* *Give an explicit counter-example to illustrate this.*

1. Every sequence has at least one convergent subsequence.
2. Let (an) be a bounded sequence and let *c* be a constant. Then:

lim sup can = c lim sup an.

1. Let (an) be a Cauchy sequence of irrational numbers. Then lim an is irrational.
2. Let {cn} be a subsequence of {an} that is strictly increasing and let {bn} be a subsequence of {an} that is strictly decreasing. Then {an} is divergent.
3. Suppose that the sequence $\left\{\frac{a\_{n}}{n}\right\}$ converges. Then {an} converges.
4. Let {an} be a sequence. Suppose that |an+2 – an+1| < |an+1 – an| for all n∈**N**. Then {an} is a Cauchy sequence.
5. If {an} diverges then the sequence {sin(an)} must diverge.
6. Suppose that {an} and {bn} are sequences satisfying 0 ≤ an ≤ bn for all n∈**N**. Then if {an} diverges it must follow that {bn} diverges.
7. If two positive series converge, then  diverges.
8. If is a convergent positive series, then  converges.
9. A sequence {an} cannot have a countable number of cluster points.
10. If an → 0 then converges.
11. If the two series and each diverge then diverges.
12. Let *A* and *B* be non-empty bounded sets. Then A∪B is bounded and

sup (A∪B) = sup A + sup B.

1. Let {an} and {bn} be bounded sequences.

Then {anbn} is bounded and lim sup anbn = (lim sup an )(lim sup bn ).

1. Let *A* be a non-empty bounded set. Then inf A < sup A.
2. Let {an} and {bn} be two sequences satisfying an < bn for all n ≥ 0 and bn – an →0 .

Let In = (an, bn). Then  consists of a single point.

**Part C**

*Instructions:**Select any 3 of the following 4 problems. You may answer all 4 to obtain extra credit.*

1. A sequence has the property that |an – an+1| ≤ 1/n! for n >>1. Prove that {an} is Cauchy.
2. Consider the proof of the theorem that *every Cauchy sequence {an} converges*:

First, one shows that {an} is bounded. Secondly, one shows that there exists a convergent subsequence $\left\{a\_{n\_{j}}\right\} of \left\{a\_{n} \right\}$. Let $L=lim⁡a\_{n\_{j}}$.

Finally one must show that {an} converges to *L*.

Give the *proof of this last result*.

1. Given a conditionally convergent series, , prove that the series,  formed from its positive terms, diverges.
2. Consider the proof of the Completeness Property for sets of real numbers:

Let *S* be non-empty and bounded above.

We construct a sequence of nested intervals In = [an, bn] satisfying: In contains at least one point of *S*, bn is an upper bound of *S* and bn – an → 0.

Invoking the Nested Intervals Theorem, there exists a point *c* such that lim an = c and lim bn = c.

*Prove that c = sup S*.

**Part D**

*Instructions:**Select any 4 of the following 6 problems. You may answer 5 to obtain extra credit but do not try to solve all 6. To receive credit, you must show your work!*

1. Let *A* be *B* be two bounded and non-empty subsets of R.

Define A + B = {a+b| a∈A, b∈B}. Prove that sup(A + B) ≤ sup A + sup B.

1. For each of the two given series, investigate convergence or divergence:



 

1. For each of the two given series, investigate convergence (conditional or absolute) or divergence:





1. Let {an} and {bn} be positive sequences and assume that  and each converge.

Prove that converges. (*Hint:* Use the Comparison Test.)

1. Let be a convergent positive series. Prove that converges also.

*Hint:* Use the geometric-mean arithmetic mean inequality.

1. Let *S* be a non-empty bounded set of real numbers and let  = sup S. Prove that

inf{ – x | x ∈S} = 0

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