**MATH 351 Old TEST 1 February 2018**

**Part A: Definitions** *[10 points each]*Be precise and careful.

1. State the *Completeness property* of the real numbers
2. Define *cluster point* of a sequence.
3. State the *Nested Intervals theorem*.
4. Define $\lim\_{n\to \infty }a\_{n}=L.$
5. State the *Limit Location Theorem.*

**Part B: *True or False****[5 points each]*

Determine if each of the following statements is *True* or *False*. If False, provide a *precise* *counter-example*; if True, give a *very brief* justification.

1. If {an} is bounded above, then the sequence {bn} defined by bn = -an is bounded below.
2. If {an} and {bn} are bounded below, then {anbn} is bounded below.
3. If {an2} converges to 9 and {an3} converges to 27, then {an} converges to 3.
4. If {an + bn} converges and {an – bn} converges, then {an} converges.
5. If {anbn} converges and {bn} converges, then {an} converges.
6. If {|an|} converges then {an} converges.
7. If {an} is decreasing and {bn} is decreasing, then {anbn} is decreasing.
8. Consider the sequence, {an}, defined recursively:



Then an ≤ 3 for all n ≥ 1.

(i) Every sequence has at least one convergent subsequence.

(j) If {an + bn} converges and {an} diverges then {bn} converges.

 (k) Let {an} be a sequence satisfying $a\_{n}>5 for n\gg 1. $

If {an} converges to L, then L > 5.

1. Suppose that {an} and {bn} are sequences satisfying 0 ≤ an ≤ bn for all n$ \in $ **N**. Then, if {an} diverges, it follows that {bn} diverges.
2. Let {cn} be a subsequence of {an} that is *strictly* increasing and let {bn} be a subsequence of {an} that is *strictly* decreasing. Then {an} is divergent.
3. Suppose that the sequence {an /n} converges. Then {an} converges.

**Part C: Proofs** *[12 points each]*

*Instructions:**Select any 4 of the following 6 problems. You may answer more than 4 to earn extra credit.*

1. Prove that every convergent sequence is bounded.

2. State and prove the *Sequence Location Theorem.*

3. For n ≥ 1, define the sequence {cn} as follows:



Determine L = lim cn. Prove that {cn} converges to L.

[*Advice:* Sketch the integrand for several values of *n*.]

4. State and prove the *Product Theorem for Limits*.

*Advice:* Use the Error-form Principle.

5. Determine $lim⁡\left(\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+…+\frac{1}{2n}\right)$ and prove it.

*Hint:* Use an area argument to estimate the sum of

$\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+…+\frac{1}{2n}$ , followed by the Squeeze Theorem.

1. Assume that $a\_{n}>0 for all n, and that \frac{a\_{n+1}}{a\_{n}}\rightarrow L where L<1.$
2. Explain why L $\geq 0.$
3. Prove that $\left\{a\_{n}\right\} is decreasing $for n>>1.
4. Prove that $a\_{n}\rightarrow 0.$