**MATH 351 Solutions: TEST 1 1 October 2018**

**Part A: Definitions & Statements of Theorems**

*[10 points each]*Be precise and careful.

1. Carefully state the **Quotient Theorem** for sequences.

Let be sequences and assume that

Then provided that

1. State the **general triangle inequality**. Aka (the extended triangle inequality).
2. Let

Define

1. State the ***Subsequence Theorem****.*

*Let*

1. State the ***Nested Intervals Theorem***.

*Let be a sequence of nested interals, that is, for all n, and assume that*

*Then there exists a unique number L Moverover, .*

1. Define

  *means that*

1. State the ***Sequence Location Theorem****.*

*Then*

**Part B: *True or False*** *[6 points each]*

Determine if each of the following statements is *True* or *False*. If False, provide a *precise* *counter-example*; if True, give a *very brief* justification.

1. If {an} and {bn} are sequences that are each bounded below, then so is the sequence {cn} defined by

cn = an + bn

**TRUE:** *Since {an} and {bn} are sequences that are each bounded below,*

*Hence*

1. Let {an} and {bn} be sequences such that {an + bn} converges. Then the sequences {an} and {bn} each converge.

**FALSE:**

*Let Clearly fails to converge..*

1. Let {an} and {bn} be sequences, such that the sequence {an + bn} diverges. Then either {an } or {bn} (or possibly both) diverges.

**TRUE:**

*The contrapositive is: If both {an} and {bn} converge, then {an + bn} converges. This follows from the limit law for sums.*

1. Let {an} and {bn} be sequences for which each of {2an + 3bn} and {4an – 5bn} converges. Then the sequences {an} and {bn} each converge.

**TRUE:** *Note that*

So .

We can use a similar argument to show that converges.

1. If {an} converges to 0, then {|an|} converges to 0.

**TRUE:** *Let Now.*

*Hence*

1. Suppose that {an} and {bn} are sequences satisfying 0 < an < bn for all n **Z+**. Then, if {an} diverges, it follows that {bn} diverges.

**FALSE:**  *Let {an} be the sequence, 1, 2, 1, 2, 1, 2, …*

*Let {bn} be the sequence 3, 3, 3, 3, …*

*Then 0 < an < bn for all n{an} diverges, yet {bn} converges*.

1. Consider a sequence {an} for which the sequence converges. Then {an} converges.

**FALSE:** Let. Then

1. Let {an} be a convergent sequence satisfying the condition: an < *M* for n>>1. Then

 *M*.

**FALSE:** *Let M = 1 and and yet*

1. Let {an} and {bn} be sequences such that {an + bn} converges to 0. Then {an} and {bn} are bounded.

**FALSE:** *Let Clearly nether is bounded.*

1. *Let* {an} be a sequence of positive real numbers for which . Then {an} converges.

**TRUE:**Since

From this, we can deduce that

**Part C: Proofs** *[16 points each]*

***Instructions:*** *Select any 3 of the following 5 problems. You may answer a fourth question to earn extra credit. Do not answer more than 4.*

**1.** Define the sequence by . Guess the limit, *L*, of and prove, *using only the definition of limit,* that converges to *L*.

***Proof:*** *Since*

*Next, Choose N\* =*

*Then*

 *since*

**2.** Prove that if the sequence

***Proof:*** *Suppose, contrary to fact, that*

Now choose

So for n>>1,

Using the triangle inequality, <

But this means that

**3.** Prove that if a > 1,

***Proof:*** *Since a > 1, a = 1 + h where h > 0.*

*Using Bernoulli’s inequality, we have an > 1 + nh > nh.*

*So, given any M > 0, choose N\* = .*

*Now, when n > N\* = , it follows that > N\* h = M.*

*Hence, by definition, .*

**4**. State and prove the *Limit Location Theorem.*

***Statement of Theorem:***

*Then*

***Proof:***  *We are given that and thatthere exists an L for which*

*Hence for all*

*Since we are given that*

*Now since*

**5.** For n ≥ 1, define the sequence {bn} as follows:

.

Prove that

***Proof:*** *.*

*Choose N\* such that*

*Then*

< (1) .

*(Here we have used the fact that is decreasing on [0, 1].)*

*Hence by the K-*