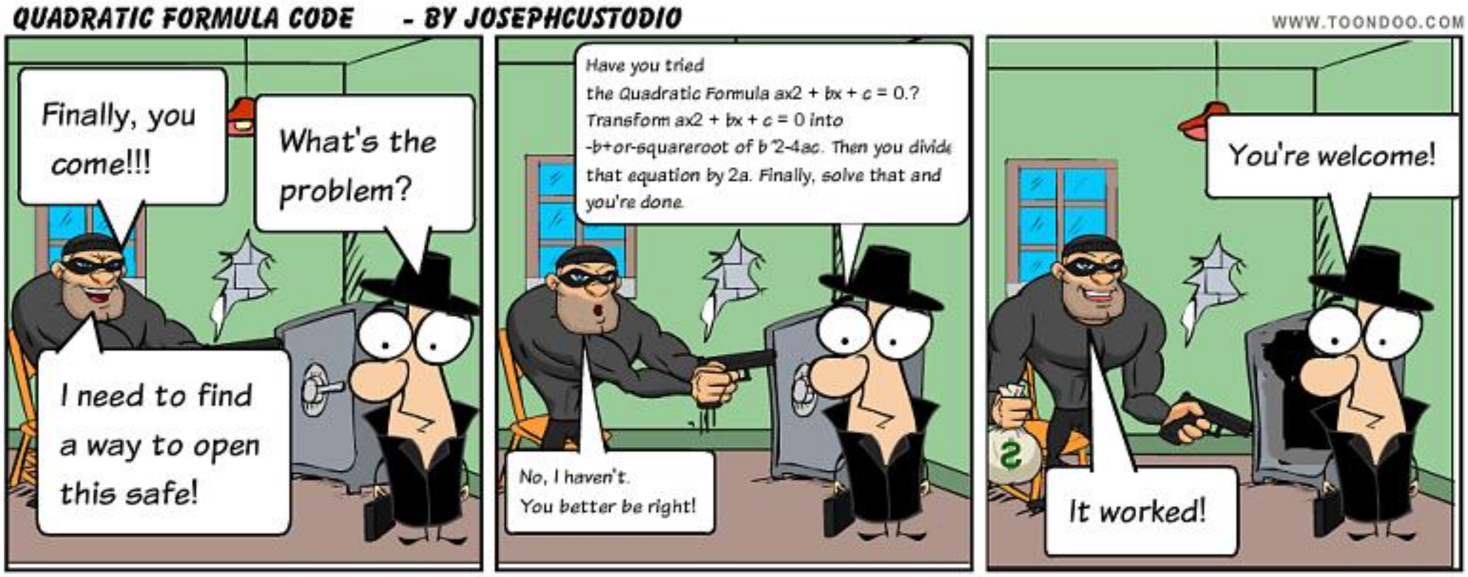
MATH 100 CLASS DISCUSSION: QUADRATICS

29 October 2019



1. Solve each of the following quadratics:
   - **(a)** $x^2 - 100 = 0$
   - **(b)** $25y^2 - 3 = 0$
   - **(c)** $4z^2 + 49 = 0$
   - **(d)** $(2t - 9)^2 = 5$
   - **(e)** $(3x + 10)^2 + 81 = 0$

2. Graph each of the following and find the vertex.
   - **(a)** $f(x) = 2(x + 3)^2 - 8$
   - **(b)** $g(x) = -(x - 2)^2 - 1$
   - **(c)** $h(x) = x^2 + 4$

3. Solve each of the following by factoring, if possible, otherwise by completing the square.
   - (a) $x^2 - 5x + 6 = 0.$
   - (b) $2x^2 + 3x - 2 = 0$
   - (c) $4x^2 + 9 = 12x$
   - (d) $x^2 - 3x - 2 = 0.$
   - (e) $4x^2 = 11x - 6$
   - (f) $x^2 - 3x - 2 = 0.$
   - (g) $x^2 - 3x - 2 = 0$
   - (h) $2x^2 - 8x + 4 = 0$
   - (i) $5x^2 + 3x + 7 = 0$
   - (j) $3x^2 - 7x + 1 = 0$

In Exercises 10 - 14, the cost and price-demand functions are given for different scenarios. For each scenario,

- Find the profit function $P(x)$.
- Find the number of items which need to be sold in order to maximize profit.
- Find the maximum profit.
- Find the price to charge per item in order to maximize profit.
- Find and interpret break-even points.

10. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, $0 \leq x \leq 15$.
11. The cost, in dollars, to produce x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is $C(x) = 10x + 100$, $x \geq 0$ and the price-demand function, in dollars per bottle, is $p(x) = 35 - x$, $0 \leq x \leq 35$.
12. The cost, in cents, to produce x cups of Mountain Thunder Lemonade at Junior's Lemonade Stand is $C(x) = 18x + 240$, $x \geq 0$ and the price-demand function, in cents per cup, is $p(x) = 90 - 3x$, $0 \leq x \leq 30$.
13. The daily cost, in dollars, to produce x Sasquatch Berry Pies is $C(x) = 3x + 36$, $x \geq 0$ and the price-demand function, in dollars per pie, is $p(x) = 12 - 0.5x$, $0 \leq x \leq 24$.
14. The monthly cost, in *hundreds* of dollars, to produce x custom built electric scooters is $C(x) = 20x + 1000$, $x \geq 0$ and the price-demand function, in *hundreds* of dollars per scooter, is $p(x) = 140 - 2x$, $0 \leq x \leq 70$.

15. The International Silver Strings Submarine Band holds a bake sale each year to fund their trip to the National Sasquatch Convention. It has been determined that the cost in dollars of baking x cookies is $C(x) = 0.1x + 25$ and that the demand function for their cookies is $p = 10 - .01x$. How many cookies should they bake in order to maximize their profit?

16. Using data from Bureau of Transportation Statistics, the average fuel economy F in miles per gallon for passenger cars in the US can be modeled by $F(t) = -0.0076t^2 + 0.45t + 16$, $0 \leq t \leq 28$, where t is the number of years since 1980. Find and interpret the coordinates of the vertex of the graph of $y = F(t)$.

17. The temperature T, in degrees Fahrenheit, t hours after 6 AM is given by:

$$T(t) = -\frac{1}{2}t^2 + 8t + 32, \quad 0 \leq t \leq 12$$

What is the warmest temperature of the day? When does this happen?

18. Suppose $C(x) = x^2 - 10x + 27$ represents the costs, in *hundreds*, to produce x *thousand* pens. How many pens should be produced to minimize the cost? What is this minimum cost?

19. Skippy wishes to plant a vegetable garden along one side of his house. In his garage, he found 32 linear feet of fencing. Since one side of the garden will border the house, Skippy doesn't need fencing along that side. What are the dimensions of the garden which will maximize the area of the garden? What is the maximum area of the garden?

20. In the situation of Example 2.3.4, Donnie has a nightmare that one of his alpaca herd fell into the river and drowned. To avoid this, he wants to move his rectangular pasture *away* from the river. This means that all four sides of the pasture require fencing. If the total amount of fencing available is still 200 linear feet, what dimensions maximize the area of the pasture now? What is the maximum area? Assuming an average alpaca requires 25 square feet of pasture, how many alpaca can he raise now?

21. What is the largest rectangular area one can enclose with 14 inches of string?

22. The height of an object dropped from the roof of an eight story building is modeled by $h(t) = -16t^2 + 64$, $0 \leq t \leq 2$. Here, h is the height of the object off the ground, in feet, t seconds after the object is dropped. How long before the object hits the ground?

23. The height h in feet of a model rocket above the ground t seconds after lift-off is given by $h(t) = -5t^2 + 100t$, for $0 \leq t \leq 20$. When does the rocket reach its maximum height above the ground? What is its maximum height?

24. Carl's friend Jason participates in the Highland Games. In one event, the hammer throw, the height h in feet of the hammer above the ground t seconds after Jason lets it go is modeled by $h(t) = -16t^2 + 22.08t + 6$. What is the hammer's maximum height? What is the hammer's total time in the air? Round your answers to two decimal places.

25. Assuming no air resistance or forces other than the Earth's gravity, the height above the ground at time t of a falling object is given by $s(t) = -4.9t^2 + v_0t + s_0$ where s is in meters, t is in seconds, v_0 is the object's initial velocity in meters per second and s_0 is its initial position in meters.

    (a) What is the applied domain of this function?

(b) Discuss with your classmates what each of $v_0 > 0$, $v_0 = 0$ and $v_0 < 0$ would mean.

(c) Come up with a scenario in which $s_0 < 0$.

(d) Let's say a slingshot is used to shoot a marble straight up from the ground ($s_0 = 0$) with an initial velocity of 15 meters per second. What is the marble's maximum height above the ground? At what time will it hit the ground?

(e) Now shoot the marble from the top of a tower which is 25 meters tall. When does it hit the ground?

(f) What would the height function be if instead of shooting the marble up off of the tower, you were to shoot it straight DOWN from the top of the tower?

26. The two towers of a suspension bridge are 400 feet apart. The parabolic cable attached to the tops of the towers is 10 feet above the point on the bridge deck that is midway between the towers. If the towers are 100 feet tall, find the height of the cable directly above a point of the bridge deck that is 50 feet to the right of the left-hand tower.

27. Discuss the meaning of the **discriminant** of a quadratic.

University of Michigan Precalculus problems:

A local grocery store sells dry goods in bulk, and one of the goods it sells is quinoa. It costs the store $110.50 per month (for the space, employee time, etc.) to be able to stock and sell quinoa and $1.25 per pound to purchase its supply of quinoa. The store charges customers $4.50 per pound for quinoa.

(a) Let C(q) be the monthly cost, in dollars, for the store to stock and sell *q* pounds of quinoa per month. Find a formula for C(q).

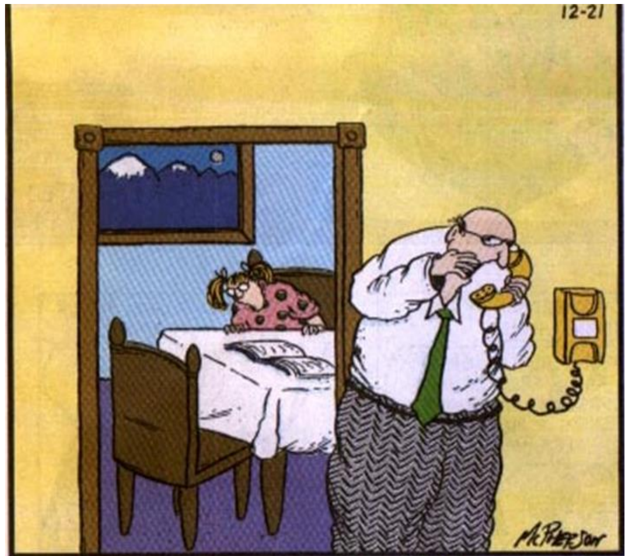
Answer: C(q) = ________________

(b) Let R(q) be the store's monthly revenue from quinoa, in dollars, if it sells *q* pounds of quinoa that month. Find a formula for R(q). Recall that revenue is the total amount of money that the store brings in, i.e. how much money customers pay.
Answer: R(q) = _________________.

(c) Assume that the store sells all of the quinoa that it buys each month. How many pounds of quinoa must the store sell in a month in order to not lose money from selling quinoa? (That is, how many pounds of quinoa must the store sell in order to break even on quinoa?) Remember to show your work.

Answer: ___________________

(d) The store also sells almonds. Suppose it sells, on average, a_0 pounds of almonds per month. Let P(a) be the profit, in dollars, that the store earns each month from selling *a* pounds of almonds. Give a *practical interpretation* of the quantity $P(a_0 + 100) - P(a_0)$. (Include units. Your interpretation should not include any math symbols or variables.)



"Uh, yeah, Homework Help Line? I need to have you explain the quadratic equation in roughly the amount of time it takes to get a cup of coffee."