

Section 1.4 Functions and change

1 Find the average rate of change of  $g(x) = (x - 2)^2 + 3$  on the following intervals:

- (a) Between 0 and 3
- (b) Between -1 and 4.

2 For the termite colony in Table 1.3,

- (a) What is the change in the population during the last 6 months? During months 6 to 9?
- (b) What is the average rate of change of the population during the last 6 months? During months 6 to 9?

Table 1.3 Population of a colony of termites

$t$ (months)	0	3	6	9	12
$P = f(t)$	1000	2500	4000	7000	2800

4 You have \$10.00 to spend on  $n$  bottles of soda, costing \$1.50 each. Are the following expressions? Equations? Give an interpretation of each expression

- (a)  $1.50n$
- (b)  $1.50n = 6.00$
- (c)  $10 - 1.50n$
- (d)  $10 - 1.50n = 2.50$

3 A car has traveled  $F(t)$  miles  $t$  hours after it starts a journey. Interpret the following statements in terms of the journey of the car. Include units.

- (a)  $F(a + 5) - F(a) = 315$
- (b)  $\frac{F(a+5) - F(a)}{5} = 63$

1. The population, in people, of a city,  $P = f(t)$ , is a function of the number of years,  $t$ , since 2010.

ANSWER ⊕

WORKED SOLUTION ⊕

2. The number of gallons of gas in a car,  $g = f(m)$ , is a function of the number of miles driven,  $m$ .

3. The number of smartphones,  $N = f(p)$ , purchased is a function of the price  $p$ , in dollars, of the smartphone.

ANSWER ⊕

4. The cost,  $C = f(w)$ , in dollars of buying a chemical is a function of the weight bought,  $w$ , in pounds.

■ In Exercises 5-7, let  $g(t)$  give the market value (in \$1000s) of a house in year  $t$ . What does the statement say about the house?

5.  $g(5) - g(0) = 30$

ANSWER ⊕

WORKED SOLUTION ⊕

6.  $\frac{g(10) - g(4)}{10 - 4} = 3$

7.  $\frac{g(20) - g(12)}{20 - 12} = -1$

■ Find the average rate of change of  $g(x) = 2x^3 - 3x^2$  on the interval in Problems 15–18.

15. Between 1 and 3.

ANSWER ⊕

16. Between  $-1$  and  $4$ .

17. Between 0 and 10.

ANSWER ⊕

WORKED SOLUTION ⊕

18. Between  $-0.1$  and  $0.1$ .

24. The graph of  $P = f(t)$  in Figure 1.16 gives the population of a town, in thousands, after  $t$  years.

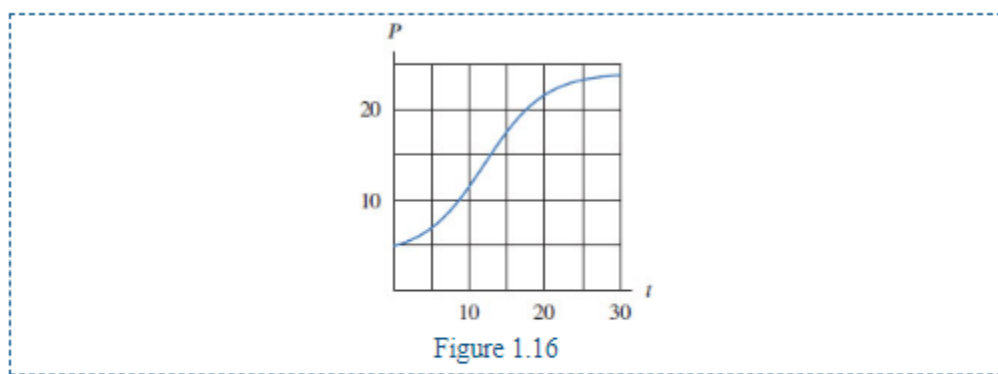


Figure 1.16

- (a) Find the average rate of change of the population of the town during the first 10 years.
- (b) Does the population of the town grow more between  $t = 5$  and  $t = 10$  years, or between  $t = 15$  and  $t = 30$  years? Explain.
- (c) Does the population of the town grow faster between  $t = 5$  and  $t = 10$  years, or between  $t = 15$  and  $t = 30$  years? Explain.

## Section 1.5 Functions, modeling, proportionality

1 A car gets 25 miles to the gallon.

(a) How far does the car travel on 1 gallon of gas? 2 gallons? 10 gallons? 20 gallons?

(b) Express the distance,  $d$  miles, traveled as a function of the number of gallons,  $g$ , of gas used. Explain why  $d$  is proportional to  $g$  with constant of proportionality  $k$ .

2 Vincent pays five times as much for a car as Dominic. Dominic pays \$300 sales tax. How much sales tax does Vincent pay (assuming they pay the same price for the car)?

3 For the same car as in Example 1:

(a) How many gallons of gas are needed for a trip of 5 miles? 10 miles? 100 miles?

(b) Find  $g$ , the number of gallons needed as a function of  $d$ , the number of miles traveled. Explain why  $g$  is proportional to  $d$  and how the constant of proportionality  $k$  is related to the constant of proportionality  $k$  in Example 1.

4 Does the function represent a direct proportionality? If so, give the constant of proportionality,  $k$ .

(a)  $f(x) = 19x$

(b)  $g(x) = x/53$

(c)  $F(a) = 2a + 5a$

(d)  $u(t) = (\sqrt{5})t$

(e)  $A(n) = n\pi^2$

(f)  $P(t) = 2 + 5t$

5 A student at a college earns \$80 for 10 hours of work. Express her earnings as a function of the number of hours worked. What is the constant of proportionality?

6 A person's heart mass is known to be proportional to his or her body mass.<sup>12</sup>

(a) A person with a body mass of 70 kilograms has a heart mass of 0.42 kilograms. Find the constant of proportionality,  $k$ .

(b) Estimate the heart mass of a person with a body mass of 60 kilograms.

7 In Example 5, the amount of money earned  $E$ , in dollars, by a student who works  $t$  hours is given by

$$E = f(t) = 8t.$$

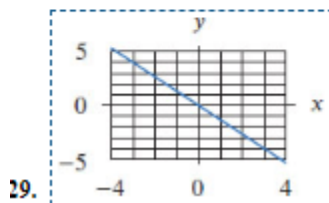
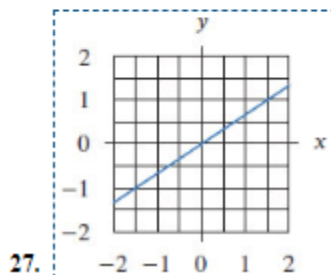
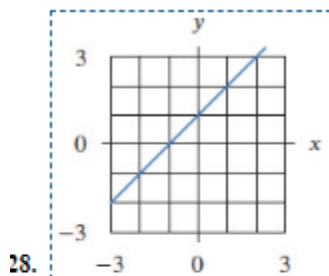
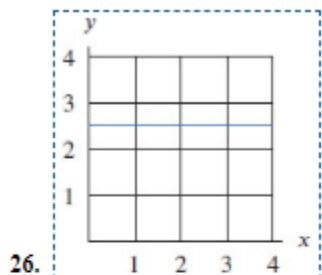
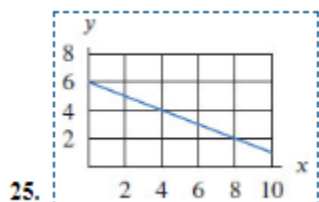
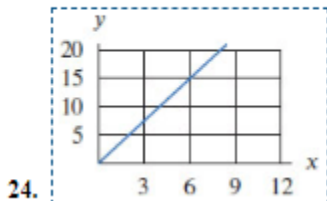
(a) What is the average rate of change of the money earned by the student

(i) Between 10 and 20 hours?

(ii) Between 6 and 8 hours?

(b) Compare your answers in part (a) to the constant of proportionality. Use the algebraic structure of the formula for  $f(t)$  to explain why this happens.

■ In Problems 24–32, determine if the graph defines a direct proportion. If it does, estimate the constant of proportionality.



42. The distance  $M$ , in inches, between two points on a map is proportional to the actual distance  $d$ , in miles, between the two corresponding locations.
- (a) If  $1/2$  inch represents 5 miles, find the constant of proportionality and give its units.
  - (b) Write a formula expression  $M$  as a function of  $d$ .
  - (c) How far apart are two towns if the distance between them on the map is 3.25 inches?

43. The blood mass of a mammal is proportional to the body mass. A rhinoceros with body weight 3000 kg has blood mass of 150 kg.

Find a formula for the blood mass of a mammal in terms of the body mass and estimate the blood mass with body mass 70 kg.

45. The *data rate* of an Internet connection is the rate in bytes per second that data, such as a web page, image, or music file, can be transmitted across the connection.

■ In Problems 49-54, put the functions in the form  $Q = kt$  and state the value of  $k$ .

49.  $Q = \frac{t}{4}$

50.  $Q = t(a + 1)$

51.  $Q = bt + ct$

52.  $Q = \frac{1}{2}t\sqrt{3}$

53.  $Q = \frac{at - bt}{c}$

54.  $Q = (t - 3)(t + 3) - (t + 9)(t - 1)$