Section 1.3 Functions and Equations

In the following equations, find a solution without performing any manipulations.

1. $10 - y = 13$
2. $\frac{w}{4} = \frac{7}{4}$
3. $\sqrt{x+1} = 7$
4. $7 + z^2 = 7$

In each of the following explain why there is no solution:

5. $\sqrt{x+8} = -4$
6. $\sqrt{x+1} = 7$
7. $-5x^2 = 13$
8. $\frac{3}{x+1} = 0$

PROBLEMS

■ In Problems 17-20, is the value of the variable a solution to the equation?

17. $t + 3 = t^2 + 9,\ t = 3$

ANSWER ⊕

WORKED SOLUTION ⊕

18. $x + 3 = x^2 - 9,\ x = -3$

19. $\frac{a+3}{a-3} = 1,\ a = 0$

ANSWER ⊕

20. $\frac{3+a}{3-a} = 1,\ a = 0$

■ In Problems 21-22, use the graph of $y = v(x)$ in Figure 1.12.

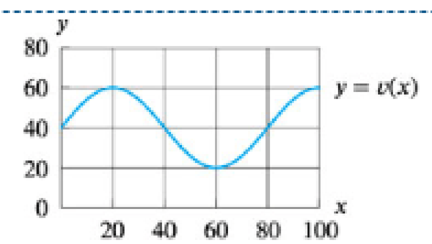

Figure 1.12

21. Solve $v(x) = 60$.

ANSWER

WORKED SOLUTION

22. Evaluate $v(60)$.

23. The tuition C, in dollars, for a semester at a small public university t years from now is given by

$$C = 3000 + 100t.$$

(a) Using the graph of C shown in Figure 1.13, estimate how many years it will take for tuition to reach $3700.

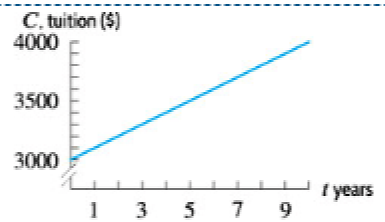

Figure 1.13

(b) Check your answer to part (a) by substituting it into the equation

$$3000 + 100t = 3700.$$

24. If a company sells s software packages, its profit per package P, in dollars, is given by

$$P = 10{,}000 - \frac{100{,}000}{s}.$$

(a) Using the graph of P shown in Figure 1.14, estimate the number of packages sold when profits per package are $8000.

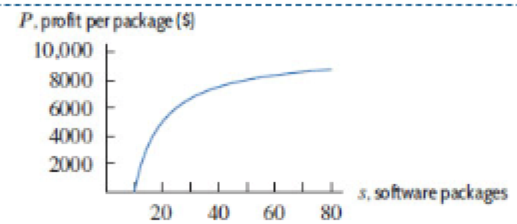

Figure 1.14

(b) Check your answer to part (a) by substituting it into the equation

$$10{,}000 - \frac{100{,}000}{s} = 8000.$$

Solve the equations in Problems 25-30.[5]

25. $3z = 22$

ANSWER

WORKED SOLUTION

26. $5x + 12 = 90$

27. $10 - 2x = 60$

ANSWER

28. $3(x - 5) = 12$

29. $\frac{x+2}{5} = 10$

ANSWER

WORKED SOLUTION

30. $2x + 5 = 4x - 9$

31. Scott developed the following solution to the equation $2(x + 3) = 8$.

$$\begin{aligned} 2(x+3) &= 8 \\ 2x + 6 &= 8 \\ 2x &= 2 \\ x &= 1. \end{aligned}$$

Describe an alternate first step that could have been used to arrive at the same solution.

32. The number of gallons of gas, g, in a car's tank, d miles after stopping for gas, is given by

$$g = 15 - d/20.$$

(a) Write an equation whose solution is the number of miles it takes for the amount of gas in the tank to reach 10 gallons.

(b) Make a plot of the gallons left for $d = 40, 60, 80, 100, 120, 140$, and indicate the solution $m = 100$ to the equation in part (a).

33. A town's population P, in thousands, t years after its incorporation is given by the function $P = 30 + 2t$.

(a) Write an equation whose solution is when the town's population reaches 50,000.

ANSWER

WORKED SOLUTION

(b) Solve the equation in part (a) by graphing both sides on the same axes.

ANSWER

WORKED SOLUTION

(c) Check your answer by solving the equation algebraically.

In Problems 61-68, does the equation have a solution? Explain how you know without solving it.

61. $2x - 3 = 7$

ANSWER

WORKED SOLUTION

62. $x^2 + 3 = 7$

63. $\frac{3x^2}{3x^2-1} = 1$

ANSWER

64. $4 = 5 + x^2$

65. $\frac{x+3}{2x+5} = 1$

ANSWER

WORKED SOLUTION

66. $\frac{x+3}{5+x} = 1$

67. $\frac{x+3}{2x+6} = 1$

ANSWER

68. $\frac{a+1}{2a} = \frac{1}{2}$

Section 1.4 Functions and change

1. The population, in people, of a city, $P = f(t)$, is a function of the number of years, t, since 2010.

ANSWER

WORKED SOLUTION

2. The number of gallons of gas in a car, $g = f(m)$, is a function of the number of miles driven, m.

3. The number of smartphones, $N = f(p)$, purchased is a function of the price p, in dollars, of the smartphone.

ANSWER

4. The cost, $C = f(w)$, in dollars of buying a chemical is a function of the weight bought, w, in pounds.

In Exercises 5-7, let $g(t)$ give the market value (in \$1000s) of a house in year t. What does the statement say about the house?

5. $g(5) - g(0) = 30$

ANSWER

WORKED SOLUTION

6. $\frac{g(10)-g(4)}{10-4} = 3$

7. $\frac{g(20)-g(12)}{20-12} = -1$

Find the average rate of change of $g(x) = 2x^3 - 3x^2$ on the interval in Problems 15-18.

15. Between 1 and 3.

ANSWER ⊕

16. Between -1 and 4.

17. Between 0 and 10.

ANSWER ⊕

WORKED SOLUTION ⊕

18. Between -0.1 and 0.1.

24. The graph of $P = f(t)$ in Figure 1.16 gives the population of a town, in thousands, after t years.

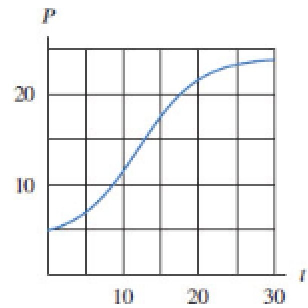

Figure 1.16

(a) Find the average rate of change of the population of the town during the first 10 years.

(b) Does the population of the town grow more between $t = 5$ and $t = 10$ years, or between $t = 15$ and $t = 30$ years? Explain.

(c) Does the population of the town grow faster between $t = 5$ and $t = 10$ years, or between $t = 15$ and $t = 30$ years? Explain.