MATH 100 PRACTICE TEST III 21 NOVEMBER 2019

NOTE: ONLY AN INEXPENSIVE NON-GRAPHING CALCULATOR IS PERMITTED!

1. **TRUE OR FALSE:** Are the statements in Problems (a) − (q) true or false? You needn't justify your answers. Write in full either "True" or "False" rather than abbreviating.

(a) The quadratic function f(x) = x(x+2) is in factored form. _____

(b) If g(x) = (x +1)(x+2), then the zeros of *g* are 1 and 2. _____

(c) A quadratic function whose graph is concave up has a maximum.

(d) The conjugate of $5 - \sqrt{19}\ is - 5 + \sqrt{19}$

(e) If the discriminant of a quadratic equals 0 then there are two roots._____

(f) If the height above the ground of an object at time *t* is given by $s(t) = at^2 + bt + c$, then s(0) tells us *when* the object hits the ground. _____

(g) To find the zeros of $f(x) = ax^2 + bx + c$, solve the equation $ax^2 + bx + c = 0$ for x. _____

(h) Every quadratic equation has two real solutions. _____

(i) There is only one quadratic function with zeros at x = −2 and x = 2. _____

(j) A quadratic function has exactly two zeros. _____

(k) The graph of every quadratic function is a parabola. _____

(l) The maximum or minimum point of a parabola is called its vertex. _____

(m) If a parabola is concave up its vertex is a maximum point. _____

(n) If the equation of a parabola is written as $y = a(x - h)^2 + k$, then the vertex is located at the point (−h, k). _____

(o) If the equation of a parabola is written as $y = a(x - h)^2 + k$, then the axis of symmetry is found at x = h. _____

(p) If the equation of a parabola is $y = ax^2 + bx + c$ and a < 0, then the parabola is concave down.

(q) A parabola cannot intersect the x-axis three times. ________

(r) A parabola has one and only one y-intercept. ________

(s) A parabola may have no x-incepts

2. Solve for *x* by *completing the square. If there is no solution, write NS:*

(a) $x^2 - 7x + 4 = 0$

(b) $\frac{x}{x+3} = \frac{3x+2}{x+1}$

(c) $5x^2 - 14x + 99 = 0$

3. Solve for x by factoring

(a) $5x^2 - 23x + 12 = 0$

(b) $3x^2 = 15 - 4x^2$

(c) $25x^2 = 5x + 6$

4. Find the value of

(a) $8^{-\frac{2}{3}}$ (b) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ (c) $\left(\frac{32}{243}\right)^{-\frac{7}{5}}$

5. Simplify and express with positive exponents:

(a) $\frac{a^{-4}b^3c^{-9}d^2}{(abcd)^2}$

(b) $\left(\frac{a^{-\frac{7}{2}}}{4c^2}\right)^{-2}$

(c) $\frac{\left(x^{\frac{1}{2}}y^{-\frac{1}{2}}\right)^{\frac{4}{3}}}{(x^2y^{-1})^{\frac{1}{3}}}$

6. Solve the equation using the quadratic formula; $9x + 4x^2 = 3$

7. Find the equation of a quadratic function that has roots x = -1 and x = 7, and passes through the point Q = (3, 1).

8. Find the *domain* of each of the following functions:

(a) $y = \frac{x+3}{x+4}$

(b) $g(x) = 3\sqrt{15-2x} + 43\sqrt{x-1}$

(c) $h(x) = \frac{x^2}{(x^2+1)(x-1)^7(x-13)^8} + (x^2 - 4x + 2019)^{\frac{1}{5}}$

9. Simplify each of the following; express with positive exponents only.

(a) $3(ab)^4c^2 + 15a^3bc - (3a^3b)^2c^4$

(b) $\frac{a^{-3}(4b)^2c^{-9}}{28ab^3c^{-4}}$

10. Where do the following parabolas intersect?

(a) $y = 2 - x^2$ and $y = x^2 - 2$.

(b)

$f(x) = 2x^2 - 4x + 7$ and $g(x) = -x^2 - x + 13.$

11. In honor of a favorite video game, a group of students decides to build a huge slingshot at a location near Madonna della Strada chapel. They will then launch a variety of large toy stuffed animals. The first "passenger" is a large stuffed panda. The height of the panda above the ground (measured in feet) *t* seconds after it is launched from the slingshot is $P(t) = -16t^2 + 48t + 8$.

(a) How long is the flying stuffed panda in the air before it lands on the lake? (Show your work and give your answer in exact form or rounded to three decimal places.)

(b) Use the method of completing the square to rewrite the formula for P(t) in vertex form. (Carefully show your work step-by-step) before it lands on the ground.

(c) After how many seconds does the flying stuffed panda reach its maximum height above the ground? What is that maximum height?

(d) In the context of this problem, what are the domain and range of P(t)? (Use either inequalities or interval notation to give your answers.)

12. Omega Ltd manufactures helmets for skiing.

If Omega charges q dollars per helmet, it finds that it can sell 1500−3q of them.

Each helmet costs $5 to produce, and the factory has a monthly rent of $400.

(a) Express the revenue, R(x), as a function of price.

(b) Express the cost, C(q), as a function of price.

(c) Express the profit, P(x), which is revenue minus cost, as a function of price.

13. The profit (in thousands of dollars) a company makes from selling a certain item depends on the price of the item and is equal to $-2p^2 + 24p - 54$.
Determine the optimal cost and the maximum profit.

14. Solve the hidden quadratic
$y^4 - 10y^2 + 9 = 0$

15. A ball dropped into a hole reaches a depth $d = 4.9t^2$ meters, where t is the time in seconds since it was dropped.
(a) Identify the coefficient and exponent of this power function.
(b) How deep is the ball after 2 seconds?
(c) If the ball hits the bottom of the hole after 4 seconds, how deep is the hole?

16.

The volume for a cylinder with radius r and height h is

$$V = \pi r^2 h.$$

If the diameter of the cylinder, w, equals the height, write V as a power function of w.

17.

Find a formula for y in terms of x.

(a) The quantity y is proportional to the fourth power of x, and $y = 150$ when $x = 2$.

(b) The quantity y is inversely proportional to the cube of x, and $y = 5$ when $x = 3$.

18. Biologists estimate that the number of animal species of a certain body length is inversely proportional to the square of the body length.[9] Write a formula for the number of animal species, N, of a certain body length in terms of the length, L. Are there more species at large lengths or at small lengths? Explain.

19. A square of side x has area x^2. By what factor does the area change if the length is
(a) doubled? (b) tripled? (c) halved? (d) multiplied by 0.1?

20. A cube of side x has volume x^3. By what factor does the volume change if the length is
(a) doubled? (b) tripled? (c) halved? (d) multiplied by 0.1?

21. A cube of side x has volume x^3. By what factor does the volume change if the side length is
(a) doubled? (b) tripled? (c) Halved? (d) multiplied by 0.1?

22. (a) If the radius of a circle is halved what happens to its area?
(b) If the side length of a cube is increased by 10%, what happens to its surface area?

23. For each of the following functions, find the range:

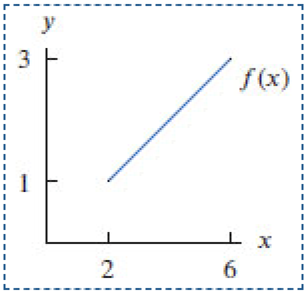
(a) $h(x) = 5 - x$

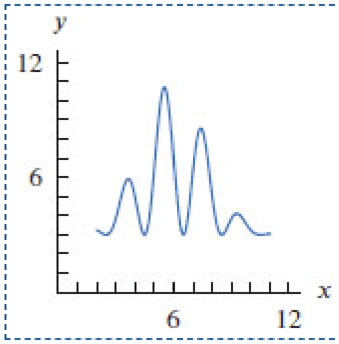
(b) $f(t) = 3t + 5$

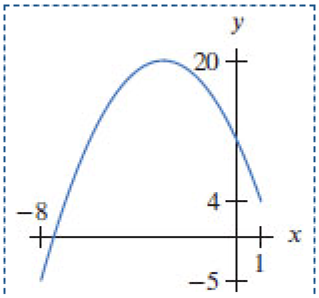
(c) $g(x) = 5$

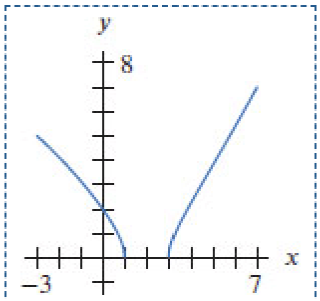
24. In Exercises 14-17, assume the entire graph is shown. Estimate:

(a) The domain
(b) The range









25. The value, V, of a car that is x years old is given by $V=f(x)=18{,}000-3000x$. Find and interpret:

(a) The domain

(b) The range

26. .

If $f(x)=5x+1$ and $g(x)=x^2-4$, find

(a) $f(2)$

(b) $f(a)$

(c) $f(a-2)$

(d) $f(x+3)$

(e) $f(g(x))$

27. Solve: (a) $\sqrt{y-2}=11$ (b) $\sqrt{3x-2}+1=10$ (c) $(x+1)^2+4=2911$

34. $\sqrt{r^2+144}=13$

35.

If $f(x)=5x+1$ and $g(x)=x^2-4$, find

(a) $f(2)$

(b) $f(a)$

(c) $f(a-2)$

(d) $f(x+3)$

(e) $f(g(x))$

36. *Without solving*, determine the *number of roots* that each of the following polynomials has. Show your work.

(a) $y = 2x^2 – 3x + 11$

(b) $y = x^2 – x – 5$

(c) $y = 318 – 30x – x^2$

(d) $y = x^2 – 100x + 2500$

37. The temperature *T*, in degrees Fahrenheit, *t* hours after 8 am is given by:

$$T(t) = -\frac{1}{2}t^2 + 12\,t + 38$$

Use appropriate units in each of your answers.

(a) What is the temperature, in degrees Fahrenheit, at 2 pm?

(b) What is the temperature, in degrees Fahrenheit, at 8 am the next day?

(c) When is the temperature the greatest? (Use am or pm.)

(d) What is the *warmest* temperature of the day? (Use appropriate units.)

38. Tristan was a giraffe. He was six feet tall when he was born, and from that moment, he grew at a constant rate of three inches per month until he was twenty feet tall, at which point he stopped growing. He remained twenty feet tall for the rest of his life. Recall that there are 12 inches in a foot and 12 months in a year. Use appropriate units in each of your answers.

(a) Let *m* be Tristan's age, in months, and let *h* be Tristan's height, in feet. Find a formula for *h* in terms of *m* that is valid during the time he was growing, that is, from the time Tristan was born until the time he reached his full-grown height of 20 feet.

(b) How old was Tristan when he stopped growing, that is, when he reached his full-grown height? Include units.

(c) Let j(m) be Tristan's height in feet when he was *m* months old. So h = j(m). Note that j(m) is defined only while Tristan is alive. Tristan died at the age of 400 months. What are the domain and range of j(m) in the context of this problem?
Use either interval notation or inequalities to give your answers.

39.

For the functions $y = (x - r)(x - s)$ graphed in Figure 3.5, give possible values for the constants r and s, and write formulas for the functions in factored form (if possible).