

(only basic calculator permitted)

To obtain any credit, you must show your work for each problem! Place a box around each answer.

1. [6 pts] Solve for t : $2\{3 - 4(5 - 6t)\} = 206$

Solution: Dividing each side by 2:

$$3 + 4(5 - 6t) = -85$$

$$4(5 - 6t) = -88$$

$$5 - 6t = -22$$

$$27 = 6t$$

$$t = 27/6 = \mathbf{9/2}.$$

2. [6 pts] Let $g(x) = 9 - x^2$

Find and *simplify* each of the following

(a) $g(-1)$

Solution: $g(-1) = 9 - (-1)^2 = 9 - 1 = \mathbf{8}$

(b) $g(3 + x)$

Solution:

$$g(3 + x) = 9 - (3 + x)^2 = 9 - (9 + 6x + x^2) = 9 - 9 - 6x - x^2 = \mathbf{-6x - x^2}$$

(c) $g(\sqrt{x})$

Solution: $g(\sqrt{x}) = 9 - (\sqrt{x})^2 = \mathbf{9 - x}$

3. [6 pts] Global temperatures may increase by up to 12°F between 2010 and 2100. Find the *average rate of change* in the global temperature between 2010 and 2100. Use appropriate units!

Solution:

The average rate of change of temperature between 2010 and 2100 =

$$\frac{\text{change in temperature}}{\text{change in time}} = \frac{12}{90} = \frac{2}{15} \text{ deg F/year}$$

4. [6 pts] The required cooling capacity, in BTUs, for a room air conditioner is proportional to the area of the room being cooled. A room of 280 square feet requires an air conditioner whose cooling capacity is 5600 BTUs.
- (a) What is the constant of proportionality, and what are its units?

Solution:

Let A = area of room (square feet) and let $y = f(A)$ = number of BTUs required to cool the room of area A .

Since y is directly proportional to A , there is a constant c for which $y = cA$.

Now we are told that when $A = 280$, $y = 5600$. So $5600 = c \cdot 280$.

Thus $c = 5600/280 = \mathbf{20 \text{ BTUs/ square foot}}$.

- (b) If an air conditioner has a cooling capacity of 10,000 BTUs, how large a room can it cool?

Solution: We know from part (a) that $y = 20A$.

Now when $y = 10000$, we have $10000 = 20A$. Hence $A = 10000/20 = \mathbf{500 \text{ sq ft}}$.

5. [6 pts] The depth, in inches, of the water in a leaking cauldron after t hours is given by

$$H(t) = (-0.08t + 6)^2$$

- (a) Find $H(0)$ and *interpret its meaning*.

Solution:

$$H(0) = (-0.08(0) + 6)^2 = 6^2 = \mathbf{36 \text{ inches.}}$$

This means that the initial level of water in the cauldron was 36 inches.

- (b) *Interpret the meaning of $H(75) = 0$.*

Solution:

This means that after 75 hours, the cauldron is empty.

Extra credit

[3 pts] A logician with some time to kill in a small town, Ontonagon, Michigan, decided to get a haircut. The town has only two barbers, each with her own shop. The logician glanced into one shop and saw that it was extremely untidy. The barber needed a shave, his clothes were unkempt, and his hair was poorly cut. The other shop was immaculate. The barber was freshly shaved and spotlessly dressed, his hair neatly trimmed. Why did the logician return to the first shop for his haircut? Explain!

Solution: *Since the town has only two barbers, each barber must have cut the other's hair. The logician chose the barber who had given his rival the better haircut. This would be the first barber.*

Mathematics is the language in which the laws of the universe are encoded, and without it, we would have no idea of where we are or how we got here.

- Tim Radford, British-New Zealand journalist, and science writer