Midway through the exam, Allen pulls
out a bigger brain.

There is something I don't know
    that I am supposed to know.
I don't know what it is I don't know,
    and yet am supposed to know,
and I feel I look stupid
    if I seem both not to know it
        and not know what it is I don't know.
Therefore I pretend I know it.

- R. D. Laing, **Knots**

Instructions: *For each problem, be certain to show your work! You may use your calculator, but you still must show your reasoning! Be sure to place a box around your answer.*

Choose any 11 of the following 13 problems. *You may answer more* than 11 to earn extra credit!

1.   Evaluate the expression $\dfrac{a+ab^3}{a}$ when a = 3 and b = -2.

Solution:
$$\frac{a+ab^3}{a} = \frac{a(1+b^3)}{a} = 1+b^3 = 1+(-2)^3 = 1+(-8) = -7$$

2. Find the slope of each of the following lines. Show your work.

(i)   y = 7x – 13

Solution: **slope = 7** since the given line is already in slope-intercept form.

    (ii) $x + 4y = 11$

Solution:

    Since $y = -\frac{1}{4}x + \frac{11}{4}$, we see that the slope $= -\frac{1}{4}$

    (iii) $x = 3y - 4$

Solution:

Since $y = \frac{1}{3}x + \frac{4}{3}$, the slope is $\frac{1}{3}$.

    (iv) $2(x - 1) + 5(6 - y) = 1$

Solution:

Rewriting the equation by expanding: $2x - 2 + 30 - 5y = 1.$

So $2x - 5y = -27$, and hence $y = \frac{2}{5}x + \frac{27}{5}$. So the slope is $\frac{2}{5}$

3. Solve for x in the following equation. Show your work.

$$179 - 18(x - 10) = 158 - 3(x - 17)$$

Solution:

Distributing on each side of the equation:

$179 - 18x + 180 = 158 - 3x + 51$ which implies

$15x = 179 + 180 - 158 - 51 = 150$ and so $x = \frac{150}{15} = \mathbf{10}$

4. If avocados sell for x dollars *per dozen* and grapefruits sell for y dollars *per half-dozen*, how much (in dollars) will it cost to buy 3 avocados and 5 grapefruits? (Note: Your answer will include the letters x and y.)

Solution:

Each avocado sells for x/12 dollars, and each grapefruit sells for y/6 dollars. Thus 3 avocados and 5 grapefruits cost

    3(x/12) + 5(y/6) = **x/4 + 5y/6 dollars**

5. Suppose that the number of minutes students take to complete a math exam is directly proportional to the number of questions on the exam.
Assume that it takes students 56 minutes to finish 16 questions on a math exam.

(a) Find the constant of proportionality.

Solution:

Let C be the constant of proportionality. Let T(q) be the number of minutes required to answer q questions. Then T(q) = Cq.

Letting q = 16, and using the fact that T(16) = 56, we have 56 = 16 q, and so $q = \dfrac{56}{16} = \dfrac{7}{2}$.

(b) If there are 50 questions on the exam, how long should it take for students to finish?

Solution:

Now, if q = 50, we find that C(50) = 50 C = $\dfrac{7}{2}$ **(50) = 175 minutes.**

6. Simplify fully by removing brackets. *Show every step!* Circle your final answer.
$$8(b - c) - [-\{a - b - 3(c - b + a)\}]$$

Solution:

Expanding from the innermost pair of parentheses:

$8(b - c) - [-\{a - b - 3c + 3b - 3a\}]$

Simplifying: $8(b - c) - [-\{-2a + 2b - 3c\}]$

Distributing the negative sign: $8(b - c) - [2a - 2b + 3c]$

Distributing the second negative sign: $8(b - c) - 2a + 2b - 3c$

So $8b - 8c - 2a + 2b - 3c$

Finally, gathering like terms: **-2a + 10b - 11c**

7. If the temperature on the first day of the month is 50 degrees Fahrenheit and 80 degrees by the 11th day, what was the average rate of change in temperature? (Include correct units.)

Solution: The average rate of change is $\dfrac{change\ in\ temperature}{change\ in\ time} = \dfrac{80-50}{11-1} = \mathbf{3}\ \dfrac{°F}{day}$.

8. Let $g(t)$ be the number of babies born in the city of Belleville in the year t. Using a *complete sentence*, explain (in non-mathematical language) the meaning of

$g(50) - g(5) = 5000$

Solution: g(50) – g(5) represents the number of babies born in Belleville between the years t = 5 and t = 50.

Thus g(50) – g(5) = 5000 means: **There were 5000 babies born in Belleville between the years t = 5 and t = 50.**

9. A cellular telephone company offers two plans. Plan A charges $29.95 for the first 400 minutes of use and 15 cents for each minute after that. Plan B costs $44.95 for unlimited minutes. How many total minutes of use would yield the same price under either plan?

Solution:

Let $C_A(t)$ and $C_B(t)$ denote the monthly cost (in dollars) of each plan with t minutes of use, where

$t \geq 400$.

Now $C_A(t) = 29.95 + 0.15\,(t - 400)$, and $C_B(t) = 44.95$,

Setting $C_A(t) = C_B(t)$, we have $29.95 + 0.15\,(t - 400) = 44.95$.

So $0.15\,(t - 400) = 44.95 - 29.95 = 15$.

Thus $t - 400 = \frac{15}{0.15} = 100$.   So $t = 100 + 400 = $ **500 minutes**

10. Let $f(x) = 3x^2 - x$

Find and simplify $f(x + 1) - f(x - 1)$

Solution:

$$f(x + 1) = 3(x + 1)^2 - (x + 1) = 3(x^2 + 2x + 1) - x - 1 = 3x^2 + 6x + 3 - x - 1$$
$$= 3x^2 + 5x + 2$$

And

$$f(x - 1) = 3(x - 1)^2 - (x - 1) = 3(x^2 - 2x + 1) - x + 1 = 3x^2 - 6x + 3 - x + 1$$
$$= 3x^2 - 7x + 4$$

Hence $f(x + 1) - f(x - 1) = 3x^2 + 5x + 2 - (3x^2 - 7x + 4) =$
$$3x^2 + 5x + 2 - 3x^2 + 7x - 4 = \mathbf{12x - 2}$$

11. Find an equation for a straight line that has y-intercept of 8 and x-intercept of -3. Show your work.

Solution:

We are given that the points P = (-3, 0) and Q = (0, 8) lie on the line. Thus the slope of the line is:

$\frac{change\ in\ y}{change\ in\ x} = \frac{8-0}{0-(-3)} = \frac{8}{3}$. Since we are given that the y-intercept is 8, the equation of the line, in slope-intercept form is:

$$y = \frac{8}{3}x + 8$$

12. The Center for Disease Control (CDC) calculates the Body Mass Index (BMI) by taking the person's weight in pounds, w, multiplying by 704.5, and dividing by the square of his or her height h in inches. If the result is at least 25.0, the person is considered overweight.

(a) Find the BMI of a person who weighs 140 pounds and is 5 feet 4 inches tall.

Solution: *Using the given information, the BMI is* $\frac{704.5\ weight\ in\ pounds)}{(height\ in\ inches)^2} = \frac{704.5\ (140)}{64^2} = \mathbf{24.08}$

(b) Is this person overweight according to the CDC?

Solution: *No, since 24.08 < 25.0*

13. Ultimate Landscaping charges its customers according to the formula **p(x) = 12 + 0.3 x** dollars, where x represents the number of one-hundred square foot sections of lawn they are being paid to mow.

(a) Find p(90).

Solution: *p(90) = 12 + 0.3(90) = 12 + 27 = **39 dollars***

(b) What is the meaning (in practical terms) of p(90)?

Solution: *In practical terms, the cost of mowing 900 square feet of lawn is $ 90.*
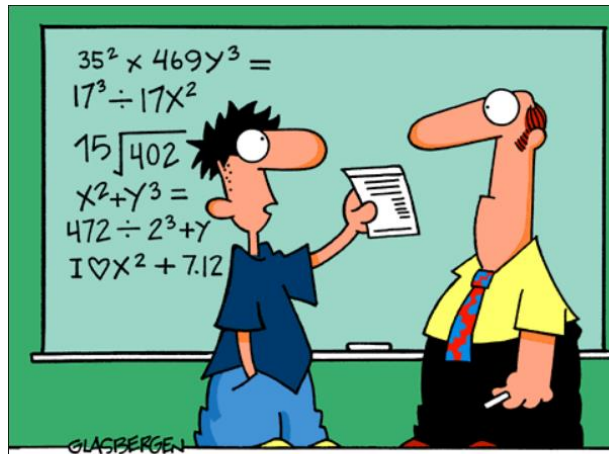
"I HAD MY DOCTOR DO A D.N.A. BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."