MATH 100 SOLUTIONS: TEST III

26 NOVEMBER 2019

1. *Without solving*, determine the *number of roots* that each of the following polynomials has. Show your work.

(a) $y = 1789 + 2019x$

number of roots = one

This is the equation of a straight line that is not horizontal.

(b) $y = 2x^2 - 7x + 11$

number of roots = none

discriminant < 0

(c) $y = x^2 (x - 1)(x - 9)$

number of roots = three

(d) $y = 41 - 30x + 29x^2$

number of roots = none

discriminant < 0

(e) $y = 4x^2 - 12x + 9$

number of roots = one

discriminant $= 0$

(f) $y = (x^2 + 5)(3x^2 + 4)$

number of roots = none

Note that $x^2 + 5 > 0$ for all x and $3x^2 + 4 > 0$ for all x.

2. Find (and simplify) the exact value of each of the following. *Do not express in decimal form.*

(a) $32^{-\frac{2}{5}}$

$$32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{\left(32^{\frac{1}{5}}\right)^2} = \frac{1}{2^2} = \mathbf{\frac{1}{4}}$$

(b) $\sqrt{\dfrac{\frac{1}{2}}{\frac{2}{16}}}$

Solution: $\sqrt{\dfrac{\frac{1}{2}}{\frac{2}{16}}} = \sqrt{\dfrac{\frac{1}{2}}{\frac{1}{8}}} = \sqrt{\frac{1}{2}\left(\frac{8}{1}\right)} = \sqrt{4} = 2$

(c) $\dfrac{3^2 4^5}{6^3}$

Solution: $\dfrac{3^2 4^5}{6^3} = \dfrac{3^2 4^5}{(2(3))^3} = \dfrac{3^2 4^5}{2^3 3^3} = \dfrac{4^5}{(2^3)\,3} = \dfrac{2^{10}}{(2^3)\,3} = \dfrac{2^7}{3} = \dfrac{128}{3}$

3. Find the ***domain*** of each of the following functions. Show your work! You need not use interval notation.

(a) $y = \frac{3}{3x+7}$

All real numbers except for $-\frac{7}{3}$.

(b) $y = \frac{2x-5}{x^2+14}$

All real numbers.

(c) $y = \frac{4-3\sqrt{x-1}}{4-x}$

All real numbers ≥ 1 *except for* $x = 4$.

(d) $y = 1 + \sqrt{x-8} + 2019\sqrt{14-x}$

Two conditions must be satisfied: $x - 8 \geq 0$ *and* $14 - x \geq 0$

This implies that $x \geq 8$ *and* $x \leq 14$. Thus, the domain is **[8, 14].**

4. Simplify each of the following; express with *positive exponents only*.

(a) $\dfrac{3(ab)^4 c^2 a^3 bc}{12(ac)^3 b}$

Solution: $\dfrac{3(ab)^4 c^2 a^3 bc}{12(ac)^3 b} = \dfrac{3a^4 b^4 c^2 a^3 bc}{12a^3 c^3 b} = \dfrac{3a^4 b^4 c^2 a^3 bc}{12a^3 c^3 b} = \dfrac{a^4 b^4 c^2 bc}{4c^3 b} =$

$\dfrac{a^4 b^4 bc}{4cb} = \boldsymbol{\dfrac{a^4 b^4}{4}}$

(b) $\frac{a^{-3}(4b)^2c^{-9}}{28ab^3c^{-4}}$

Solution:

$$\frac{a^{-3}(4b)^2c^{-9}}{28ab^3c^{-4}} = \frac{a^{-3}16b^2c^{-9}}{28ab^3c^{-4}} = \frac{16}{28a^4bc^5} = \boldsymbol{\frac{4}{7a^4bc^5}}$$

(c) $\left(\frac{15x^2y^3}{5x^3y^{-4}}\right)^{-2}$

Solution:

$$\left(\frac{15x^2y^3}{5x^3y^{-4}}\right)^{-2} = \left(\left(\frac{15x^2y^3}{5x^3y^{-4}}\right)^{-1}\right)^2 = \left(\frac{5x^3y^{-4}}{15x^2y^3}\right)^2 =$$

$$\left(\frac{x}{3y^7}\right)^2 = \boldsymbol{\frac{x^2}{9y^{14}}}$$

5. Factor fully:

(a) $7ab^4c + 28ab^3c + 14ab^2c$

Solution:

$7ab^4c + 28ab^3c + 14ab^2c = abc(b^3 + 4b^2 + 2b)$

(b) $x(x^2 - 16)$

Solution: $x(x^2 - 16) = x(x^2 - 4^2) = \boldsymbol{x(x+2)(x-2)}$

(c) $5x^2 + 17x + 6$

Solution: $5x^2 + 17x + 6 =$ **$(5x + 2)(x + 3)$**

6. (a) Determine the two points of intersection of the line $y = 2x + 6$ and the parábola $y = x^2 + 6x + 9$.

Solution: Set $2x + 6 = x^2 + 6x + 9$. Then $x^2 + 4x + 3 = (x + 3)(x + 1)$.

Now we have intersections when x = -3 and when x = -1.

When x = -3, y = 0; when x = -1, y = 4.

Hence the points of intersection of the line and the parabola are **(-3, 0) and (-1, 4)**

(b) Determine the two points of intersection of the two parabolas:

$y = 33 – x^2 – 2x$ and $y = x^2 + 2x + 3$

Solution: Set $33 – x^2 – 2x = x^2 + 2x + 3$

Then $2x^2 + 4x – 30 = 0$. Dividing by 2: $x^2 + 2x – 15 = 0$

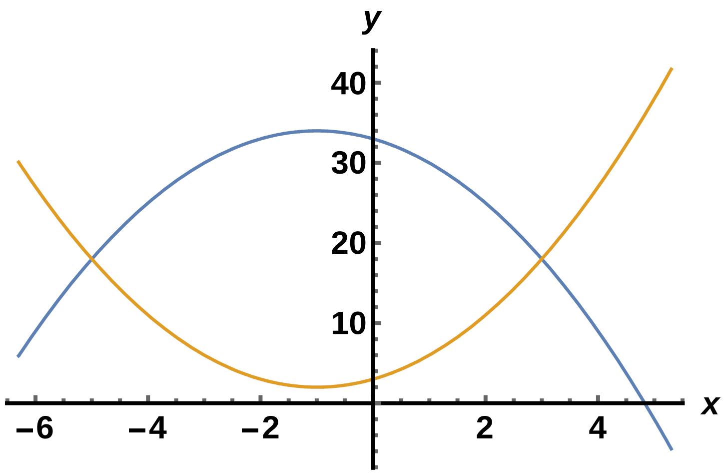
Completing the square: $(x + 1)^2 – 1 – 15 = 0$

So $(x + 1)^2 = 16$, and thus $x + 1 = \pm 4$.

$Hence\ x = -1 + 4 = 3\ or\ x = -1 - 4 = -5.$

Now, when x = 3, y = 18, and when x = -3, y = 6.

And so, the two points of intersection are **(-5, 18) and (3, 18).**



7. (a) Using the *quadratic formula*, solve the equation $2x^2 – 3x – 4 = 0$

Solution: $$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)} = \frac{\mathbf{3\pm\sqrt{41}}}{\mathbf{4}}$$

(b) Solve by completing the square. Show all work.

$x^2 + 8x – 1 = 0$

Solution: $x^2 + 8x – 1 = (x + 4)^2 - 16 - 1 = 0.$

So $(x+4)^2 = 17$
Hence $x + 4 = \pm\sqrt{17}$.
Finally, $\mathbf{x = -4 \pm \sqrt{17}}$**.**

(c) Solve by completing the square. Show all work.

$2x^2 - 4x - 3 = 0$

Solution: Begin by dividing each side by 2 to obtain $x^2 - 2x - \frac{3}{2} = 0$.
So $(x-1)^2 - 1 - \frac{3}{2} = 0$.
This implies $(x-1)^2 = \frac{5}{2}$.
Hence $x - 1 = \pm\sqrt{\frac{5}{2}}$.

Finally $\boldsymbol{x = 1 \pm \sqrt{\frac{5}{2}}}$.

(d) Solve for x. You may use any technique that you wish.

$$\frac{x}{x+2} = \frac{x+3}{x}$$

Solution:

Cross-multiply to obtain $x^2 = (x+2)(x+3) = x^2 + 5x + 6$

So $5x + 6 = 0, \textit{from which we find } \boldsymbol{x} = -\frac{\mathbf{6}}{\mathbf{5}}$.

8. The temperature *T*, in degrees Fahrenheit, *t hours after midnight* is given by:

$$T(t) = -2\,t^2 + 44\,t - 238$$

Use appropriate units in each of your answers.

(a) What is the temperature, in degrees Fahrenheit, at noon?

$$T(12) = -2(12^2) + 44(12) - 238 = \mathbf{2\ deg}\,\boldsymbol{F}.$$

(b) What is the temperature, in degrees Fahrenheit, at 8 am?

$$T(8) = -2(8^2) + 44(8) - 238 = \mathbf{-14\ deg}\,\boldsymbol{F}.$$

(c) When is the temperature the greatest? (Use am or pm.)

Solution: Since the axis of symmetry is $t = -\frac{b}{2a}$ $it\ follows\ that\ t = \frac{-44}{-4} = 11\ is\ the\ axis\ of\ symmetry$ for our graph.

Thus the temperature is maximal at **11 am**.

(d) What is the *warmest* temperature of the day? (Use appropriate units.)

Solution: The maximal temperature is achieved at 11 am. The temperature at that time is

$$T(11) = -2(11^2) + 44(11) - 238 = \mathbf{4\ deg}\ \boldsymbol{F}.$$

9. Oscar Phones Ltd, Albertine's new start-up, sells smarter phones for 700 – 0.4x dollars each, where x is the number of phones sold. The total manufacturing cost is 300 +10x dollars.

(a) Find the *revenue* function.

R(x) = x(700 – 0.4x) dollars

(b) Find the *profit* function.

P(x) = R(x) – C(x) = x(700 – 0.4x) – (300 +10x) =

$700x - 0.4x^2 - 300 - 10x = \mathbf{-0.4x^2 + 690x - 300}$

(c) *How many* smarter phones must be sold to *maximize* profits?

Solution: The vertex of this parabola occurs a $x = -\frac{b}{2a} = -\frac{690}{-(2)0.4} = 862.$

Hence, the profit is maximized when **862** phones are sold.

(d) Compute the maximum profit.

Solution: $P(862) = -0.4(862)^2 + 690(862) - 300 =$ **$ 297,262**

(e) What is the breakeven point, i.e., when is the profit equal to 0?

Solution: The breakeven point is reached when profit = 0.

Setting P(x) = 0:

$$-0.4x^2 + 690x - 300 = 0$$

Solving for x we find that the breakeven point occurs when x = **1724** phones are sold.

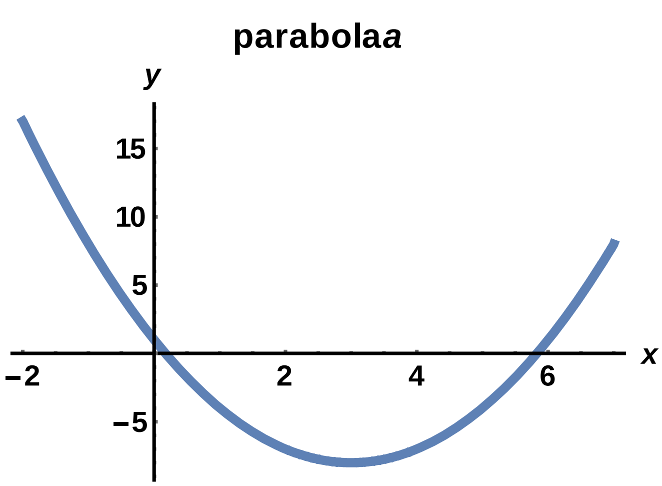
10. Consider the four graphs of parabolas below. For each graph, give *plausible* values for b, c and k for which $y = b(x - c)^2 + k$. You may assume that b = 1 or b = -1.

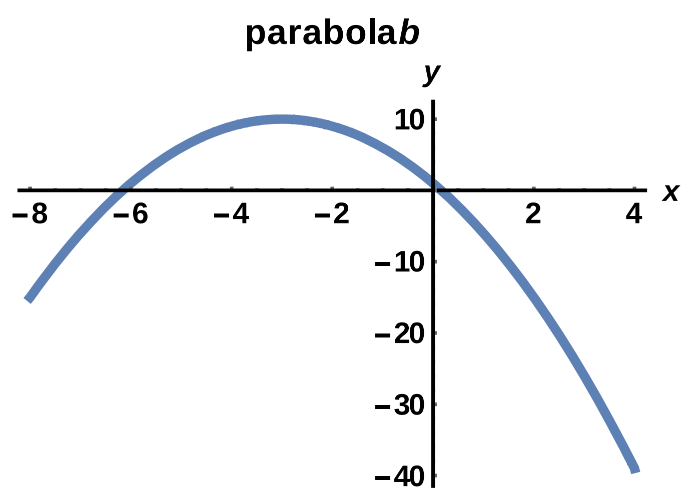
(a) $\mathbf{y = (x - 3)^2 - 8}$

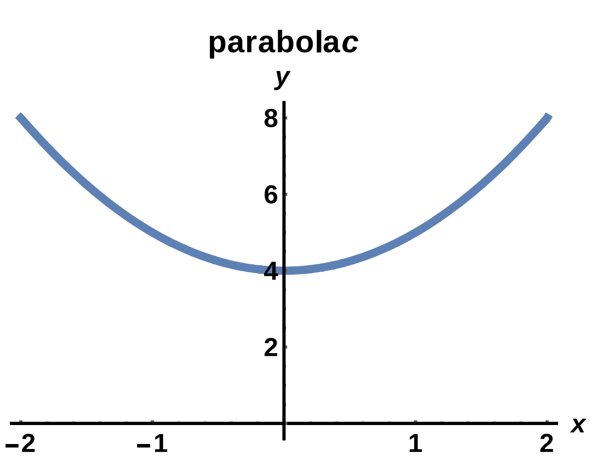
(b) $\mathbf{y = -(x + 3)^2 + 10}$

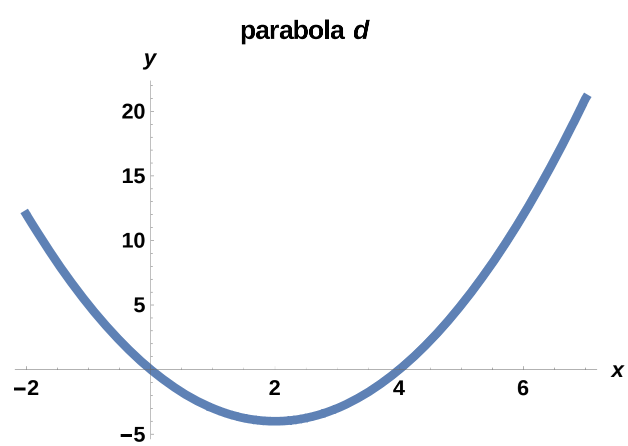
(c) $\mathbf{y = (x - 0)^2 + 4}$

(d) $\mathbf{y = (x - 2)^2 - 5}$









11. In honor of a favorite video game, a group of students decides to build a massive slingshot at a location near Madonna Della Strada chapel. They will then launch a variety of large toy stuffed animals. The first "passenger" is a giant stuffed panda. The height of the panda above the ground (measured in feet) *t* seconds after it is launched from the slingshot is $P(t) = -16t^2 + 64t + 12$.

i. Explain (without using mathematical language) the meaning of

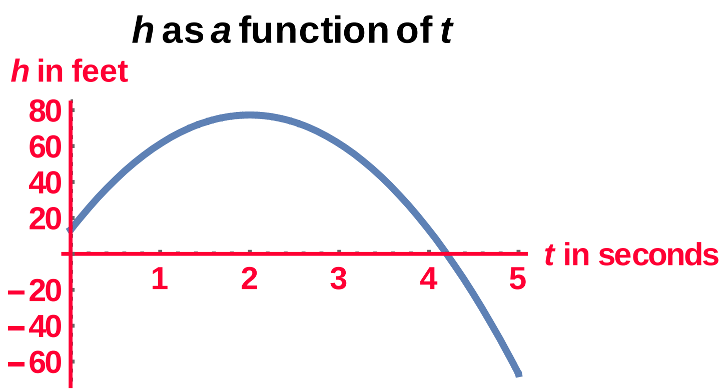
$$P(1.3) = 68.2$$

Use a complete sentence.

Answer: **At 1.3 seconds after the launch, the panda is 68.2 feet above the ground.**

ii. How long is the flying stuffed panda in the air before it lands on the lake? (Show your work and give your answer in exact form or rounded to three decimal places.)

Solution: Here is a graph of the panda's flight



The stuffed panda will reach the lake when $h(t) = 0$.

So setting $h(t) = -16t^2 + 64t + 12 = 0$, we solve for t using any method you wish and find that

$$t = \frac{1}{2}\left(4 + \sqrt{19}\right) \cong \mathbf{4.18}\ seconds\ from\ take\ off.$$

iii. Use the method of completing the square to rewrite the formula for P(t) in *vertex form*. (Carefully show your work step-by-step) before it lands on the ground.

Solution: $h(t) = -16t^2 + 64t + 12 = -16(t^2 - 4t - \frac{3}{4}) =$

$-16[(t-2)^2 - 4 - \frac{3}{4}] = \mathbf{-16(t-2)^2 + 76.}$

iv. After how many seconds does the flying stuffed panda reach its maximum height above the ground? What is that maximum height?

Solution: Maximum height is reached when $t = -\frac{b}{2a}$ = 64/32 = **2 seconds**

v. In the context of this problem, what are the domain and range of P(t)? (Use either inequalities or interval notation to give your answers.)

Solution: The domain is **[0, 4.18].**

The range is **[0, 76].**

Extra Extra Credit: Albertine, a young farmer, is trying to cross a river. She is taking with her a rabbit, carrots, and a fox, and she has a small raft. She can only bring 1 item at a time across the river because her raft can only fit either the rabbit, the carrots, or the fox. How does she cross the river? (You can assume that the fox does not eat the rabbit if Albertine is present, you can also assume that the fox and the rabbit are not trying to escape and run away.) Hint: This will involve several steps on the part of Albertine.

Solution:

First, Albertine takes the rabbit across and returns to the fox & carrots. Next, she takes the carrots, but when she arrives at the other side with the rabbit, she leaves the carrots and takes the rabbit back on the raft with her to return and get the fox. She exchanges the rabbit for the fox and returns to drop the fox off with the carrots, and finally goes back to get the rabbit.