MATH 161 CLASS DISCUSSION: 2 DECEMBER 2019

SUBSTITUTION IN INDEFINITE & DEFINITE INTEGRALS

"Runners to your mark. Get set. Go! ... OK, come get your T-shirts."

I Use the *method of substitution* to evaluate each of the following indefinite integrals.

(A) $\int x\sqrt{2x-1}\,dx$ (B) $\int x(x+1)^{\frac{1}{3}}\,dx$

(C) $\int x^2(x+1)^{2018}dx$ (D) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx$

(E) $\int \sec x\,dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}dx$

(F) $\int \tan(4x+1)\sec^2(4x+1)\,dx$ (G) $\int \frac{x}{\sqrt{1-x^4}}dx$

(H) $\int \frac{e^x}{1+e^{2x}}dx$ (I) $\int \frac{\sin z}{(3+2\cos z)^2}dz$

$(J)\quad \int \frac{t^2-5}{t+2}\,dt$

$(K)\quad \int \frac{\sqrt{t}}{\left(1+t^{\frac{3}{2}}\right)^2}\,dt$

$(L)\quad \int \frac{\sqrt{\ln x}}{x}\,dx$

$(M)\quad \int x^7\left(5+x^8\right)^5\,dx$

$(N)\quad \int \frac{\sec^2 y}{1-\tan y}\,dy$

$(O)\quad \int \left(\tan^4 u\right)\left(\sec^2 u\right)du$

$(P)\quad \int \sqrt{a+bx}\;\,dx$

$(Q)\quad \int \sqrt{3+\sqrt{x}}\;\,dx$

$(R)\quad \int \frac{1}{\sqrt{1+\sqrt{x}}}\;\,dx$

II Integrating squares of sine and cosine.

$(A)\quad \sin^2 x = \frac{1-\cos(2x)}{2}$

$(B)\quad \cos^2 x = \frac{1+\cos(2x)}{2}$

III For each of the following integrals, use the *change of variable theorem* for definite integrals:

$(A)\quad \int_0^1 x(x^2+1)^3\,dx$

$(B)\quad \int_0^{\pi/10} \sin^2(5x)\,dx$

$(C)\quad \int_0^3 \sqrt{x+5}\,dx$

$(D)\quad \int_0^4 \sqrt{1+\sqrt{x}}\,dx$

$$(E)\quad \int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$$

IV Let f(x) be continuous on the real line. Verify that

$$(A)\quad \int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$$

$$(B)\quad \int_a^b f(x+h)\,dx = \int_{a+h}^{b+h} f(x)\,dx$$

$$(C)\quad \int_{ca}^{cb} f(x)\,dx = c\int_a^b f(cx)\,dx$$

A substitute shines brightly as a king,

Until a king be by.

- Shakespeare, **The Merchant of Venice**