CLASS DISCUSSION 4 DECEMBER 2019

L' HÔPITAL'S RULE

Marquis Guillaume de l'Hôpital (1661 – 1704)

I Evaluate each of the following limits, using l'Hôpital's rule when appropriate:

$(A)\quad \lim_{x\to 0} \frac{e^{3x}-1}{x}$ $\qquad (B)\quad \lim_{x\to\infty} \frac{x^2}{e^x}$

$(C)\quad \lim_{x\to\infty} e^{-x}\sqrt{x}$ $\qquad (D)\quad \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

$(E)\quad \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$ $\qquad (F)\quad \lim_{x\to\infty} x^{1/x}$

$(G)\quad \lim_{x\to 0} \frac{\sqrt{1+x}-1-x/2}{x^2}$ $\qquad (H)\quad \lim_{x\to 0} \frac{e^x-1-x}{x^2}$

$(I)\quad \lim_{x\to 0} \frac{\cos x-1+x^2/2}{x^4}$ $\qquad (J)\quad \lim_{x\to 0} \frac{x(\cosh x-1)}{\sinh x-x}$

(K) $\lim_{x\to\infty} \dfrac{\ln(ax+b)}{\ln(cx+d)}$ (where a, b, c, d are positive constants)

(L) $\lim_{x\to 1} \dfrac{x^{2017}-1}{x-1}$ (M) $\lim_{x\to \pi/2-} \dfrac{\tan x}{\sec x+1}$

(N) $\lim_{x\to 0+} (\sin x)^x$ (O) $\lim_{x\to 1} \dfrac{\ln x}{x-1}$

(P) $\lim_{x\to 0+} x\ln x$ (Q) $\lim_{x\to\infty} \left(1+\dfrac{5}{x}\right)^x$

(R) $\lim_{x\to 0} \left(\dfrac{1}{\sin x}-\dfrac{1}{x}\right)$ (S) $\lim_{x\to 1+} x^{\frac{1}{x-1}}$

(T) $\lim_{x\to\infty} \dfrac{\displaystyle\int_1^x \dfrac{t^4+t+1}{t^2+4t+13\ln t}\,dt}{(2x+1)^3}$

(U) $\lim_{x\to 1+} \left(\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right)$

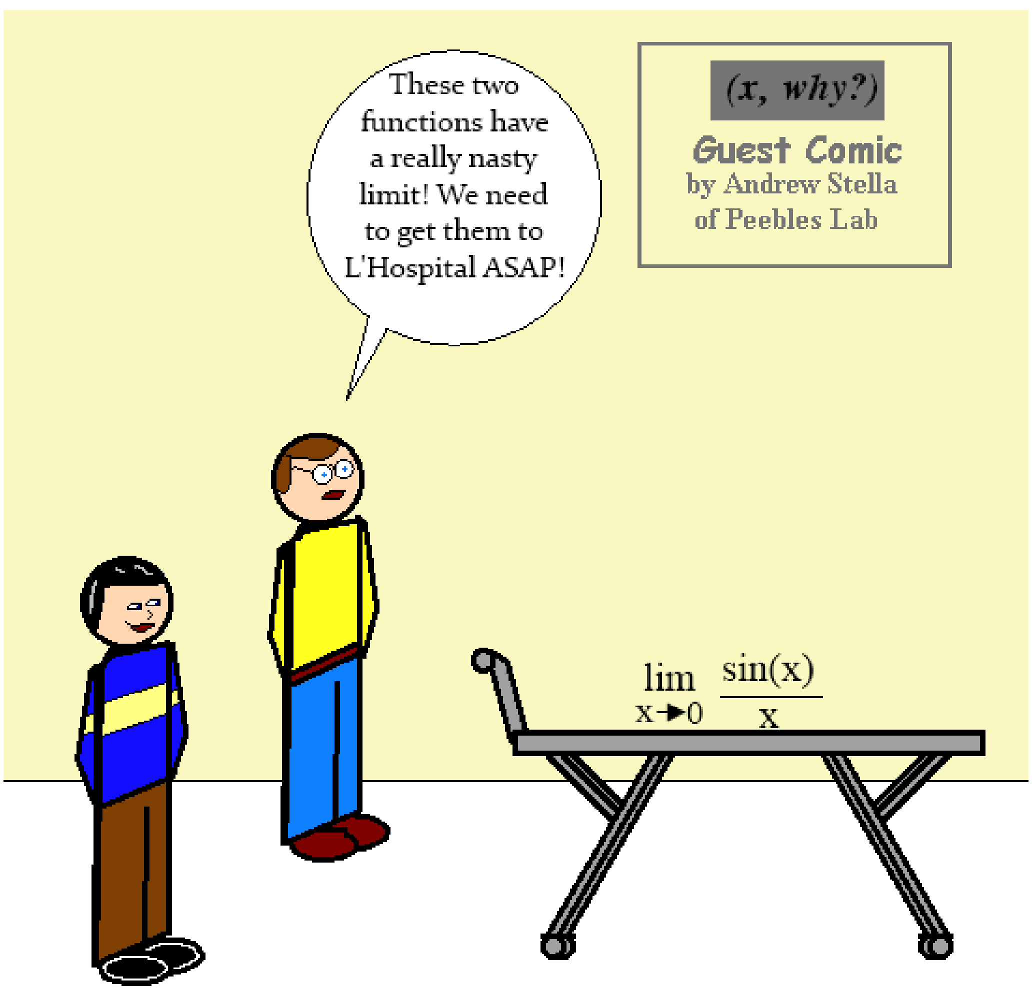
(V) Explain what happens if one tries to use l'Hopital's rule on the following: $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2-1}}$

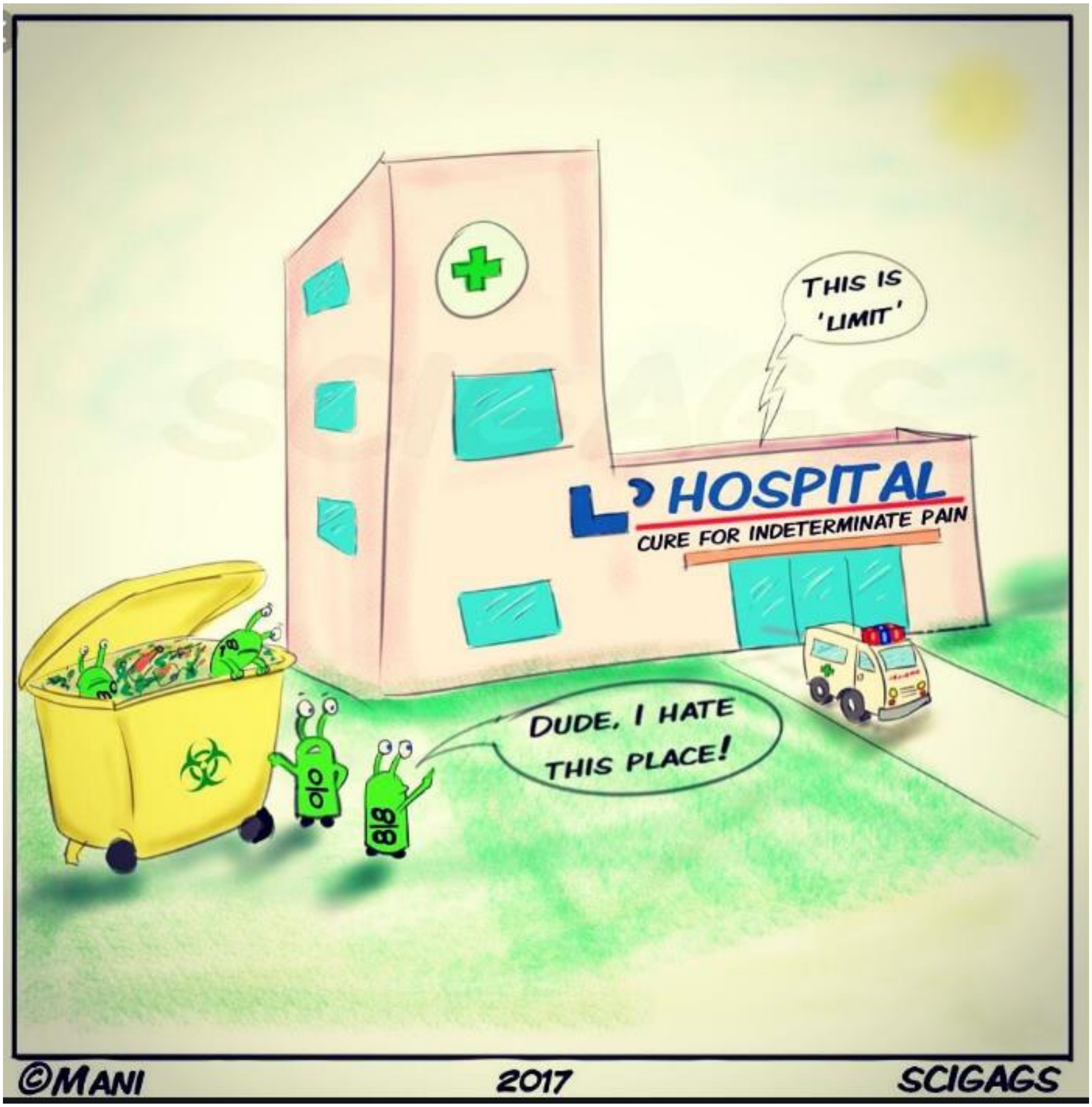
II For each of the following piecewise-defined functions, find the value for c that makes the function continuous at x = 0.

$$(A) \quad f(x) = \begin{cases} \dfrac{4x - 2\sin 2x}{2x^3} & if \quad x \neq 0 \\ c \quad if \quad x = 0 \end{cases}$$

$$(B) \quad g(x) = \begin{cases} \left(e^x + x\right)^{1/x} & if \quad x \neq 0 \\ c \quad if \quad x = 0 \end{cases}$$

III The *Gamma Function*, Γ(n), is defined in terms of the integral of the function $f(x) = x^{n-1}e^{-x}$ for $n > 0$. Show that for any fixed value of *n*, the limit of f(x) as $x \to \infty$ is 0.





The notion of infinity is our greatest friend; it is also the greatest enemy of our peace of mind.

- James Pierpont