MATH 161 CLASS DISCUSSION: 13 NOVEMBER 2019

THE RIEMANN INTEGRAL & AREA

Georg Friedrich Bernhard Riemann

(1826 – 1866)

1) Using the *area interpretation* of the Riemann integral, evaluate each of the following:

(a) $\int_0^4 x\,dx$ (b) $\int_{-2}^4 t\,dt$ (c) $\int_{-2}^{1} |x|\,dx$

(d) $\int_{-3}^{2} |3x+4|\,dx$ (e) $\int_0^1 \sqrt{1-x^2}\,dx$ (f) $\int_{-3}^{3} x^{1789}\sin(x^2+1)\,dx$ (g) $\int_0^{2\pi} \cos x\,dx$

(h) $\int_0^3 5x\,dx$ (i) $\int_0^1 x\,dx + \int_1^5 x\,dx$

2) Let g be a continuous function on the interval [-5, 5]. Suppose that

$$\int_0^5 g(x)\,dx = 4$$

Evaluate each of the following Riemann integrals:

(a) $\int_0^5 \left(g(x)+3\right)dx$

(b) $\int_{-2}^{3} g(x+2)\,dx$

(c) $\int_{-5}^{5} g(x)\,dx$ *if g is even*

(d) $\int_{-5}^{5} g(x)\,dx$ *if g is odd*

3) Find the constants a and b that *maximize* the value of the definite integral:

$$\int_a^b \left(4-x^2\right)dx$$

Justify your answer!

4) (a) Assuming that $0 \le a < b$, find a formula for $\int_a^b x\,dx$.

(b) Assuming that $a < 0 < b$, find a formula for $\int_a^b |x|\,dx$.

5) State the major properties of the Riemann integral.

6) Suppose that h is integrable and that $\int_{-1}^{1} h(x)\,dx = 0$ and $\int_{-1}^{3} h(x)\,dx = 6.$

Find:

(a) $\int_{1}^{3} h(x)\,dx$ (b) $\int_{1}^{3} \left(5h(x)+3\right)dx$

7) Suppose that f and h are integrable and that

$$\int_{1}^{9} f(x)\,dx = -1,\ \int_{7}^{9} f(x)\,dx = 5 \text{ and } \int_{7}^{9} h(x)\,dx = 4$$

Find:

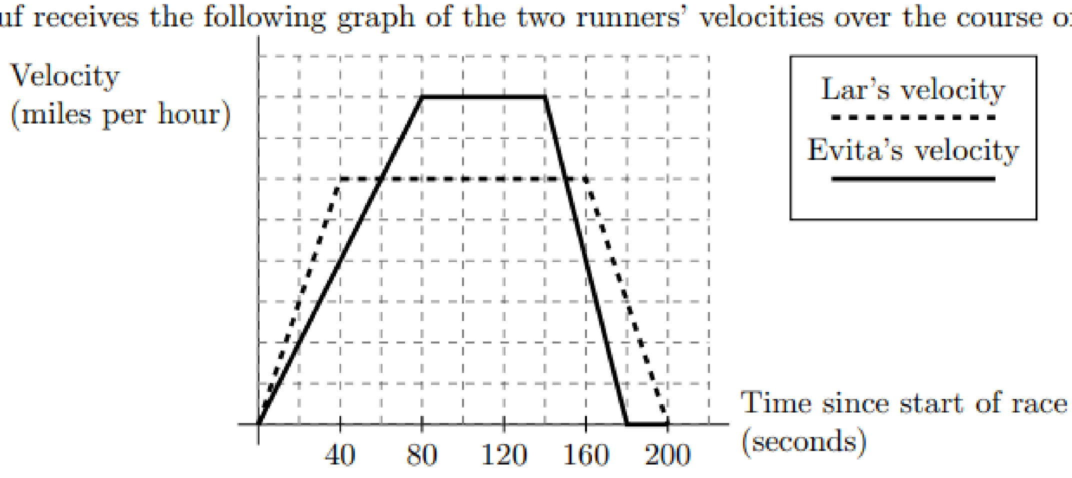
(a) $\int_{1}^{9} -3f(x)\,dx$ (b) $\int_{7}^{9} (f(x)+h(x))\,dx$

(c) $\int_{7}^{9} (5f(x)-3h(x))\,dx$ (d) $\int_{1}^{7} (f(x)-|x-4|)\,dx$

University of Michigan problems

Exercise I

[12 points] Lar Getni and Evita Vired run a half-mile race. After the race, C.T. Latnem Adnuf receives the following graph of the two runners' velocities over the course of the race.



Unfortunately, whoever made the graph forgot to label the scale of the vertical axis, and C.T. needs your help to answer the following questions. You may assume that the horizontal grid lines are evenly spaced, but do not assume that the scales of the two axes are the same. You may also assume that both runners completed the race and then stopped running.

a. [1 point] Who won the race?

b. [2 points] During what time interval(s) was Lar ahead of Evita?

c. [2 points] During what time interval(s) was Lar running faster than Evita?

d. [4 points] What was the maximum speed (in miles per hour) attained by Lar? By Evita? Remember to show your work.

e. [3 points] Let v(t) (respectively, w(t)) be Evita's (respectively, Lar's) velocity in miles per hour t seconds after the start of the race. Write an equation involving one or more integrals that expresses the following statement: N seconds after the start of the race, Evita is M miles ahead of Lar. Your answer may involve v(t) and w(t).

Exercise II

4. (9 points) Last week David ran the Naked Mile, starting out at a fast pace with the idea of winning the race. His friend, John rode along on his bike to clock David's times. However, at the end of the race the police were chasing David so that he kept on running to avoid being arrested. John followed and recorded David's speeds at 5 minute intervals. David was slowing down all the time, but fortunately for him, the policemen were unable to catch him. They finally gave up chasing him after 25 minutes. David continued for an additional five minutes before stopping.

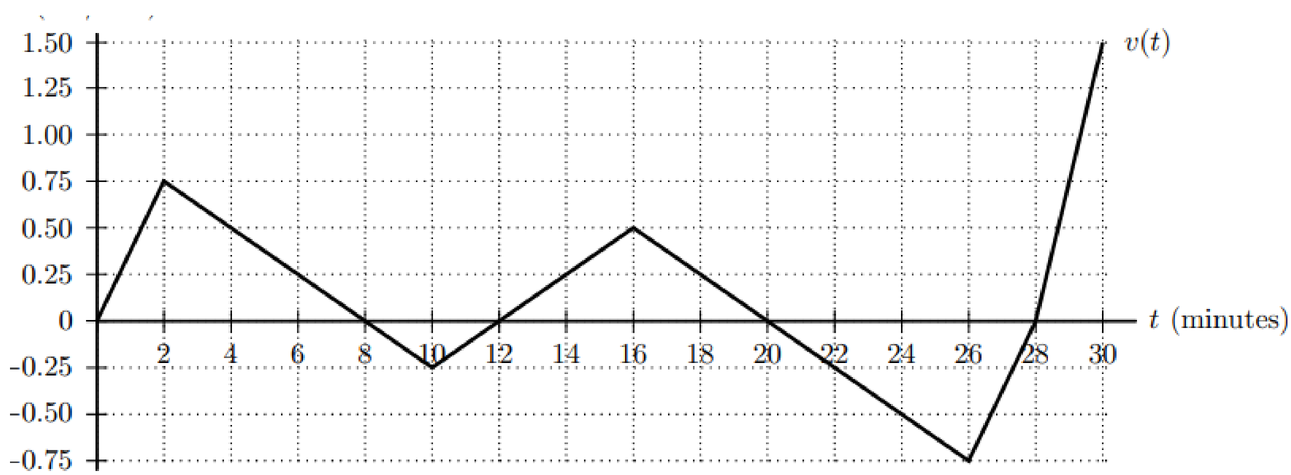
The speeds John clocked are recorded in the following table. In recounting his experience, David wondered how far he actually ran in the half hour. Help him out by answering the questions in parts (a) and (b). (Be sure to show your work when answering the questions).

time (in minutes):	0	5	10	15	20	25	30
speed (in miles per minute):	.2	.16	.14	.12	.12	.1	.05

(a) Assuming that David's speed never increases throughout the run, use the data in the table to determine the best estimate for the total distance that David ran during the 30 minutes.

Exercise III

[10 points] Unfortunately, Sebastian left the King's castle but never made it to Adam's manor because the brakes on his car were sabotaged. Sebastian was driving on a straight road between the King's castle and Adam's manor when he found himself unable to brake and racing down a hill. Let $v(t)$ be Sebastian's velocity (in kilometers per minute) t minutes after he left the King's castle. Note that $v(t)$ is positive when Sebastian is traveling towards Adam's manor. Sebastian suspected he was being followed so he occasionally backtracked.
Sebastian crashed 30 minutes into his journey. A graph of $v(t)$ is given below.



a. [3 points] How far from the King's castle was Sebastian 12 minutes into his journey? *Include units.*

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b. [2 points] What was Sebastian's average velocity during the first 12 minutes of his journey?

c. [2 points] Of the four times below, circle the one at which Sebastian's acceleration was the greatest (i.e. most positive).

$t = 6$ $\qquad\qquad$ $t = 13$ $\qquad\qquad$ $t = 20$ $\qquad\qquad$ $t = 27$

d. [3 points] In the interval $0 \le t \le 30$ when was Sebastian the closest to the King's castle? When was he the furthest from the King's castle?