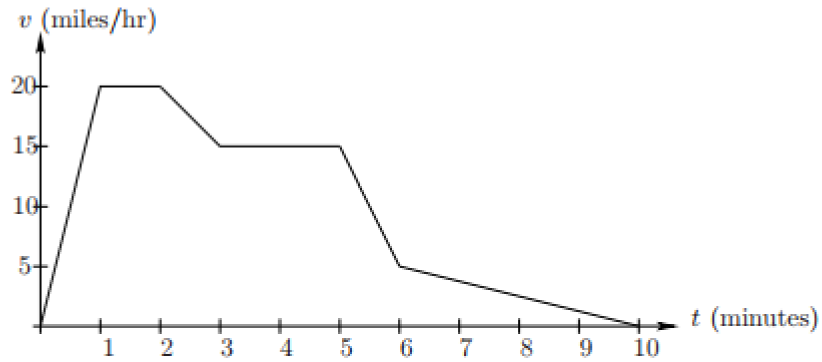
MATH 161   CLASS DISCUSSION: 18 NOVEMBER 2019

THE RIEMANN INTEGRAL & AREA, CONTINUED

1. *(University of Michigan)* Three congressional representatives leave a meeting at the White House and return home to watch the TV show, *The Simpsons*. In this episode, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner can sail through all the traffic and travels with a constant velocity of 10 miles per hour.

Note: Beware of the units, miles & minutes.



Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner can sail through all the traffic and travels with a constant velocity of 10 miles per hour.

Note: Beware of the units, miles & minutes.

(a) When (or during which interval(s) is Homer traveling at maximum speed? What is that speed?

(b) How far does Homer travel during the 10 minutes shown in the graph?

(c) A what time, $t > 0$, is Homer the greatest distance ahead of Principal Skinner?

(d) Does Principal Skinner overtake Homer, and, if so, when? Explain.

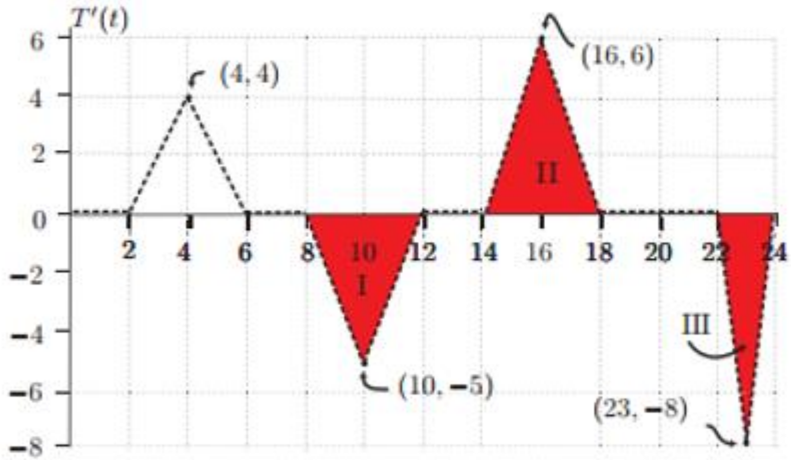
2. Suppose that H(c) is the average temperature, in degrees Fahrenheit that can be maintained in Oscar's apartment during the month of December as a function of the cost of the heating bill, *c*, in dollars.

(a) Using *complete sentences*, give a practical interpretation of each of the following:

$H(50) = 65$

$H'(50) = 2$

(b) Suppose T(t) gives the temperature (in degrees Fahrenheit) in Oscar's apartment on December 18th as a function of time, *t,* in hours since midnight. Below is a graph of $T'(t)$, *the derivative of* T.



(c) When Oscar returns home from work at 6 pm, the temperature in his apartment is 67 °F. What was the temperature when he left for work at 8 am?

(d) If the temperature at 6 pm is 67 °F, what is the minimum temperature in the apartment on December 18th?

3. A gas leak is discovered in a large municipal building. The rate at which gas is leaking into the building is increasing, as indicated in the table below.

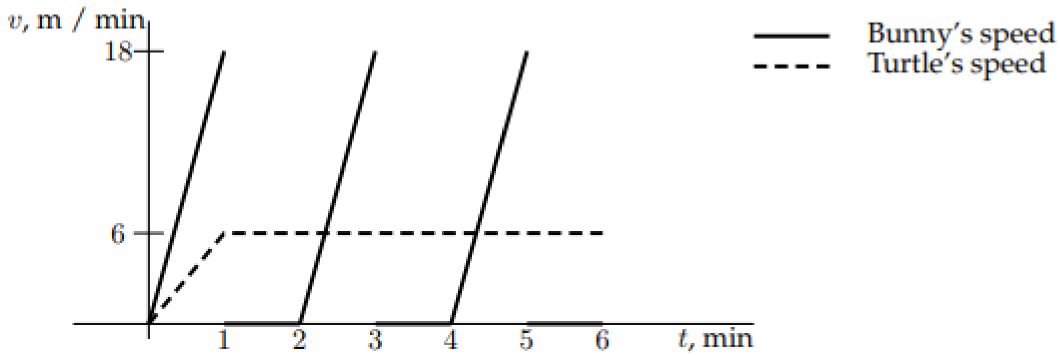
Time (hours)	0	7	14	21	28	35
Rate (grams/hour)	125	127	132	140	153	171

Initially there is no gas in the building. Suppose the building is well sealed so that no gas escapes from it.

(a) Determine lower and upper estimates for the amount of gas in the building after 21 hours.

(b) How often would the rate need to be measured over the interval from t = 0 to t = 35 in order to find upper and lower estimates within 100 grams of the actual amount of gas in the building?

4. The Awkward Turtle is competing in a race! Unfortunately his archnemesis, the Playful Bunny, is also in the running. The two employ very different approaches: the Awkward Turtle takes the first minute to accelerate to a slow and steady pace which he maintains through the remainder of the race, while the Playful Bunny spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute - and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



(a) What is the Awkward Turtle's average speed over the first two minutes of the race? What is the Playful Bunny's?

(b) The Playful Bunny immediately gets ahead of the Awkward Turtle at the start of the race. How many minutes into the race does the Awkward Turtle catch up to the Playful Bunny for the first time? Justify your answer.

(c) If the race is 60 meters total, who wins? Justify your answer