CLASS DISCUSSION: 20 NOVEMBER 2019

THE FTC

1. How is the Riemann integral defined for functions that are not necessarily positive?

2. Define ***average value*** of a function over an interval.

3. State the two versions of the ***Fundamental Theorem of Calculus***.

4. Find the *area beneath the given curve lying above the given interval:*

(a) $f(x) = x^3$ above $[1, 3]$

(b) $g(x) = \sin x$ over $[0, \pi]$

(c) $z(x) = (x-1)^2$ over $[0, 3]$

(d) $h(x) = (\ln x) / x$ over $[1, 4]$

(e) $s(t) = t^3(2 + 3t^4)^3$ over $[0, 1]$

5. For each function in (4), find the *average value* over the given interval.

6. Using the FTC, compute $g'(e)$ given that

$$g(x) = \int_0^x t^5(5 - 4\ln t)^{13}\, dt$$

7. Using the FTC compute:

$$\frac{d}{dx}\int_0^x e^{-u^2}\, du$$

8. Let $0 < k < 1$ and consider the Elliptic Integral:

$$E(x) = \int_0^x \frac{1}{\sqrt{1 - k^2 \sin^2 t}}\, dt$$

Find dE/dx.

9. Using the FTC and the Chain Rule, calculate dF/dx given that:

$$F(x) = \int_0^{\sin x} \frac{1}{1 + v^5}\, dv$$

10. Consider the function defined by:

$$H(x) = \int_{\frac{\pi}{2}}^{x^3} \cos t\, dt$$

Calculate dH/dt by:

(a) Using the FTC and the Chain Rule.

(b) By first performing the integration.

(c) Compare the two answers that you have obtained.

11. A particle is moving along a line so that its velocity is given by:
$v(t) = (t - 1)(t - 4)(t - 5) = t^3 - 10t^2 + 29t - 20$ ft/sec at time t seconds.

(a) Find the *displacement* of the particle over the time interval [1, 5].

(b) Find the *total distance* traveled by the particle over the time interval [1, 5].

12. Find the *area* of the region bounded by the graphs of the given functions. Sketch!

13. (a) $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

(b) $y = 2 - x^2$ and $y = x$

(c) $y = \sin x$ and $y = \cos x$ over $[\pi/4, 5\pi/4]$

(d) $y = 3x^3 - x^2 - 10x$ and $y = 2x - x^2$

(e) $y = x^3$ and $y = x^6$

(f) $y = x^3 - x$ and $y = 0$

(g) $y = x^2 - 4x + 3$ and $y = 3 + 2x - x^2$

14. *(University of Michigan)* Let C(t) be the temperature, in degrees Fahrenheit, of a warm can of soda t minutes after it was put in a refrigerator. Suppose C(10) = 62.

(a) Assuming C is invertible, give a practical interpretation of the statement
$C^{-1}(45) = 40$

(b) Give a practical interpretation of the statement $C'(10) = -0.4$.

(c) Give a practical interpretation of the statement $\int_0^{10} C'(t)\,dt = -5$

(d) Assuming the statements in parts (a)-(c) are true, determine C(0).

(e) What is the practical meaning of $\int_0^1 C(t)\,dt$?

- **Exercises from Oxford Math Center:**

1.

a. $\int_0^1 (x^3 + x^2)\, dx$

b. $\int_0^{\pi/2} \cos x\, dx$

c. $\int_1^2 (4x^2 - x)\, dx$

d. $\int_0^{\pi/4} \frac{1}{3}\sec^2 x\, dx$

e. $\int_2^5 \frac{x+2}{\sqrt{x-1}}\, dx$

f. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\, dx$

g. $\int_0^{\pi/4} \tan^3 x \cos x\, dx$

h. $\int_0^{\pi/3} \sec x \tan x\, dx$

i. $\int_0^{\sqrt{\pi}} x \sin(x^2)\, dx$

j. $\int_0^{\pi/3} \sin(2x)\cos^2(2x)\, dx$

k. $\int_{\pi/6}^{\pi/2} \frac{1-\sin^2 x}{\cos x}\, dx$

l. $\int_0^{4/3} \sqrt{1 + \frac{9}{4}x}\, dx$

m. $\int_0^1 x^2\sqrt{4+5x^3}\, dx$

n. $\int_0^2 \frac{1}{(3x+2)^2}\, dx$

o. $\int_0^2 \frac{4y}{\sqrt{25-4y^2}}\, dy$

p. $\int_0^1 18x\sqrt{3x^2+1}\, dx$

q. $\int_0^{\sqrt{2}/4} \frac{t}{\sqrt{1-4t^2}}\, dt$

r. $\int_4^8 \frac{3t}{\sqrt{t^2-15}}\, dt$

s. $\int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right) dx$

t. $\int_0^{\pi/4} \cos^2(2x)\sin(2x)\, dx$

u. $\int_0^{1/2} \frac{\text{Arctan}\,(2x)}{1+4x^2}\, dx$

v. $\int_0^{1/\sqrt{2}} \frac{x}{\sqrt{1-4x^4}}\, dx$

w. $\int_1^e \frac{\ln\sqrt{x}}{x}\, dx$

x. $\int_3^6 \frac{y}{3\sqrt{y^2-8}}\, dy$

2. Find the area between the graph of the given function and the x-axis over the given interval

a. $y = x^3$; $[-1, 3]$

b. $y = \sin x$; $[-\frac{\pi}{2}, \frac{\pi}{2}]$

c. $y = -x^2 + 4$; $[-4, 2]$

d. $y = x^3 - 3x^2$; $[1, 4]$

e. $y = \frac{1}{4}x^4 - x^2$; $[-1, 1]$

f. $y = 3x^4 + 4x^3$; $[-1, 1]$