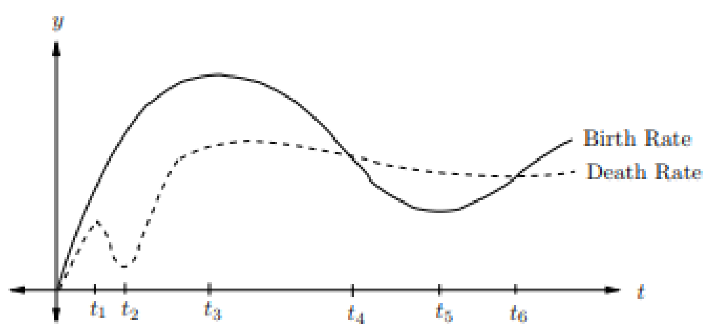
CLASS DISCUSSION: 22ND NOVEMBER

#GIVINGTUESDAY
December 3, 2019

UNIVERSITY OF MICHIGAN PROBLEMS

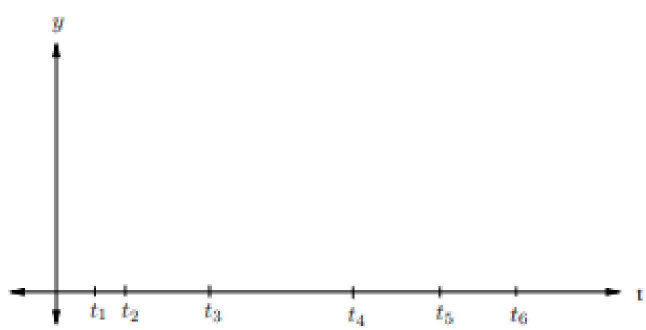
9. (8 points) Last year a local entomologist studied the birth and death rate of mosquitos in the Ann Arbor area during the month of May. His research yielded the following graph.



(a) Which of the labelled times t_1 through t_6 is the time when there were the largest number of mosquitos in Ann Arbor during May?

(b) Which of the labelled times t_1 through t_6 is the time when the quantity of mosquitos in Ann Arbor was increasing most rapidly during May?

(c) Sketch a possible graph of the number of mosquitos alive during the month of May on the axes below. Make sure to clearly indicate any maxima, minima, or inflection points.



2. [15 points] There were 3 trillion trees in the world in the year 2000.

- Since the year 2000, a group of environmentalists have recorded the number of trees lost in the world due to natural causes or due to human activities. Let $C(t)$ be the rate at which the number of trees decreases due to any of these causes, t years after the year 2000, in trillions of trees per year.
- At the same time, some governments and other organizations plant new trees to increase the number of trees in the world. The group is also measuring the rate $P(t)$ at which the trees are being planted, t years after the year 2000, in trillions of trees per year.

Throughout this question, you may assume that the functions $C(t)$ and $P(t)$ describe the only changes to the number of trees in the world.

a. [7 points] In parts (i) and (ii) below, give a mathematical expression that may involve $C(t)$, $P(t)$, their derivatives, and/or definite integrals.

(i) Find an expression for the total number of trees in the world (in trillions) in the year 2005.

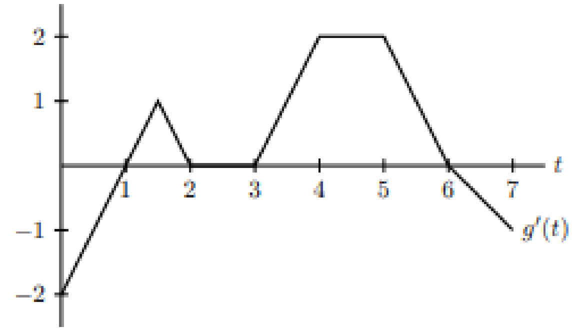
Answer: ________________________________

(ii) Find an expression for the average rate at which the trees were being planted (in trillions of trees per year) between the years 2002 and 2009.

Answer: ________________________________

b. [3 points] Write a practical interpretation of the statement $\int_{13}^{17} C(t)dt = 0.05$. Your answer must be a complete sentence.

3. [12 points] The function $g(t)$ is the volume of water in the town water tank, in thousands of gallons, t hours after 8 A.M. A graph of $g'(t)$, the **derivative** of $g(t)$, is shown below. Note that $g'(t)$ is a piecewise-linear function.



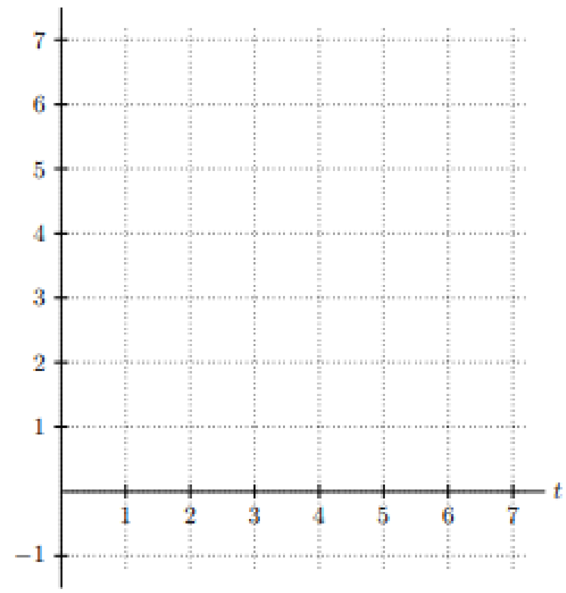
a. [4 points] Write an integral which represents the average rate of change, in thousands of gallons per hour, of the volume of water in the tank between 9 A.M. and 1 P.M. Compute the exact value of this integral.

b. [2 points] At what time does the tank have the most water in it? At what time does it have the least water?

Answer: The tank has the most water in it at ____________________.

The tank has the least water in it at ____________________.

c. [6 points] Suppose that $g(3) = 1$. Sketch a detailed graph of $g(t)$ and give both coordinates of the point on the graph at $t = 7$.



8. [7 points] Mr. R. DeVark discovers that there is a loud humming sound emanating from a tree in his backyard. The volume of the sound at any point in the yard is a function of the point's distance from the tree.

- Let $V(x)$ be the rate of change (in decibels per meter) of the volume of the sound where x is the distance (in meters) from the tree.
- Let $K(t)$ be the distance, in meters, of Mr. DeVark from the tree t seconds after he first notices the sound.

Assume that K is invertible and that V, K, and K^{-1} are differentiable.

a. [3 points] Give a practical interpretation of the equation $\int_{10}^{40} V(x)\,dx = -5$ in the context of this problem. *Remember to use a complete sentence and include units.*

b. [2 points] Which one of the following expressions represents the instantaneous rate of change (in decibels per second) of the volume at which Mr. DeVark hears the sound 30 seconds after he first notices the sound? Circle the one best answer.

$V(30)$	$V(30)K(30)$	$V'(30)K(30)$
$V'(30)$	$V(30)K'(30)$	$V'(30)K'(30)$
$V(K(30))$	$V(K(30))K(30)$	$V'(K(30))K(30)$
$V(K'(30))$		
$V'(K(30))$	$V(K(30))K'(30)$	$V'(K(30))K'(30)$
$V'(K'(30))$	$V(K'(30))K'(30)$	$V'(K'(30))K'(30)$

c. [2 points] Which of the following is the best interpretation of the equation $(K^{-1})'(15) = -2$? Circle the one best answer.

i. Between 15 and 15.5 seconds after Mr. DeVark notices the humming sound, he moves about 1 meter closer to the tree.

ii. It takes about 1 second for Mr. DeVark to go from being 15 meters away from the tree to 14.5 meters away from the tree.

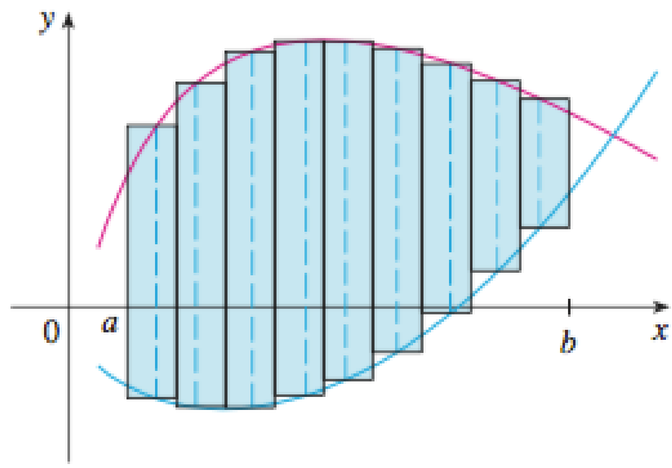
iii. The volume of the humming sound is about 1 decibel lower at a point 15.5 meters from the tree than it is at a point 15 meters from the tree.

iv. When Mr. DeVark is 15 meters away from the tree, it is about 2 seconds before he notices the humming sound

v. The volume of the humming sound Mr. DeVark hears is about 1 decibel lower 15 seconds after he first notices it than 0.5 seconds later.

vi. When Mr. DeVark is 15 meters away from the tree, he moves about 2 meters closer to the tree in the next second.

AREA BOUNDED BY CURVES



1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
2. Find the area of the region in the first quadrant bounded above by $y = x^{1/2}$ and below by the x-axis and the line $y = x - 2$.
3. Repeat exercise (2) above, but this time integrate with respect to *y*.
4. Find the area of the crescent-shaped region in the first quadrant that is bounded by $y = x^{13}$ and $y = x^{15}$.
5. Find the area of the region bounded by $y = 7 - 2x^2$ and $y = x^2 + 4$.
6. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
7. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
8. Find the area of the region enclosed by $y = x(a^2 - x^2)^{1/2}$, where $a > 0$, and $y = 0$.
9. Find the area of the region enclosed by $y = (|x|)^{1/2}$ and $5y = x + 6$.
10. Find the area of the region enclosed by $x = y^3 - y^2$ and $x = 2y$.
11. Find the area of the region bounded by $4x^2 + y = 4$ and $x^4 - y = 1$.
12. Find the area of the region enclosed by $y = 2 \sin x$ and $y = \sin (2x)$, $0 \le x \le \pi$.
13. Find the area of the region enclosed by $y = \cos(\pi x/2)$ and $y = 1 - x^2$.
14. Find the area of the region enclosed by $y = \sin(\pi x/2)$ and $y = x$.
15. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{2x}$, below by the curve $y = e^x$, and on the right by the line $x = \ln 3$.

DIFFERENTIATING INTEGRALS

Differentiate with respect to x each of the following integrals using the FTC and Leibniz's Formula:

1. $y = \int_{3}^{x} \sqrt{5+\cos^3 t}\, dt$

2. $y = \int_{1}^{x} \frac{5}{3+t^4}\, dt$

3. $y = \int_{\sec x}^{4} \frac{1}{1+t^2}\, dt$

4. $y = \int_{1/x}^{x} \frac{1}{t}\, dt$

5. $y = \int_{\cos x}^{\sin x} \frac{1}{1-t^2}\, dt$

6. $y = \int_{\sqrt{x}}^{x^2} \frac{e^t}{t}\, dt$

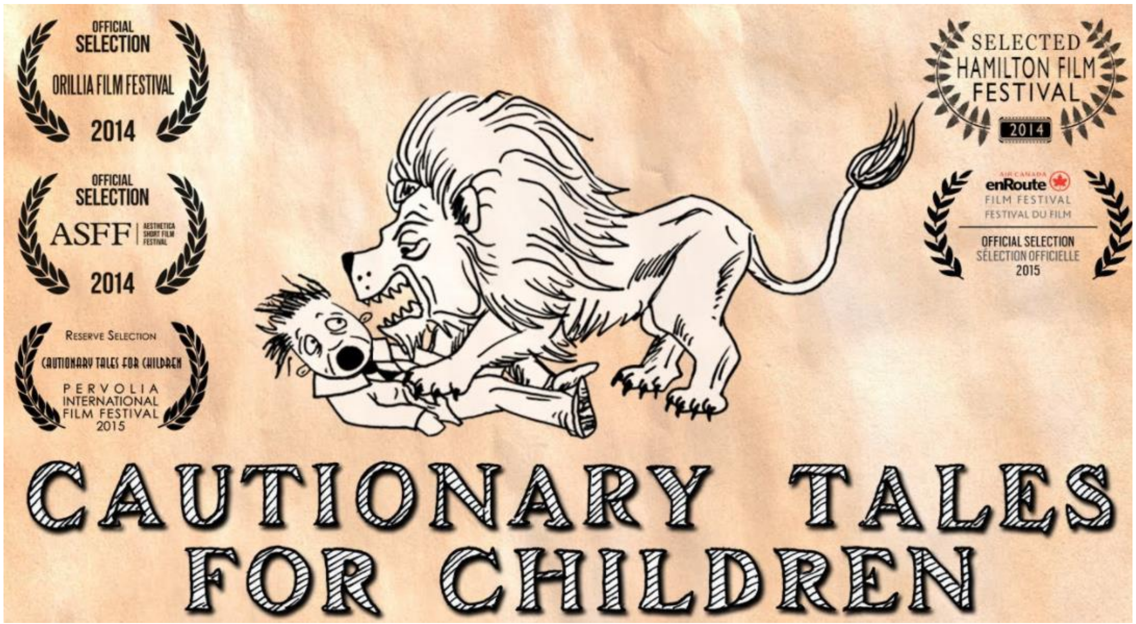
USING INTEGRALS TO APPROXIMATE RIEMANN SUMS

Evaluate each of the following limits:

1. $\lim_{n\to\infty} \frac{1^5+2^5+3^5+\ldots+n^5}{n^6}$

2. $\lim_{n\to\infty} \frac{1^3+2^3+3^3+\ldots+n^3}{n^4}$

3. $\lim_{n\to\infty} \frac{1}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\ldots+\sin\frac{n\pi}{n}\right)$



The nicest child I ever knew
Was Charles Augustus Fortescue.
He never lost his cap, or tore
His stockings or his pinafore:
In eating Bread, he made no Crumbs,
He was extremely fond of sums.

\- Hilaire Belloc, **Cautionary Tales** (1907)