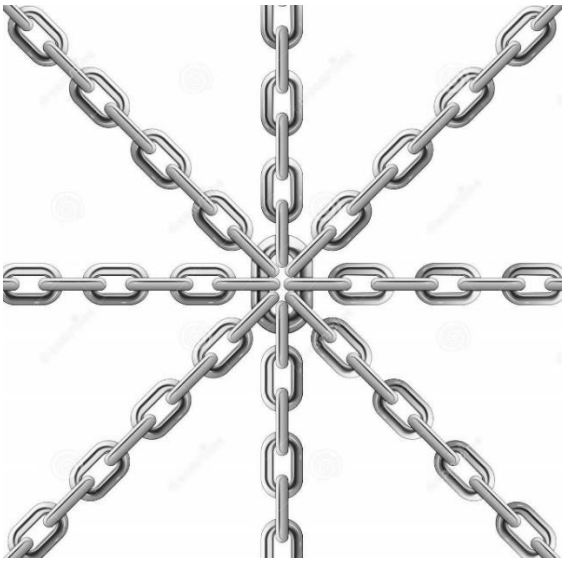
# CLASS DISCUSSION: 11 OCTOBER 2019

## Chain Rule



***Stewart exercises:***

1, 2, 3, 4, 5 and 6 Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative $dy/dx$.

1. $y = \sqrt[3]{1+4x}$
2. $y = \left(2x^3 + 5\right)^4$
3. $y = \tan \pi x$
4. $y = \sin(\cot x)$
5. $y = e^{\sqrt{x}}$
6. $y = \sqrt{2 - e^x}$

Compute *the derivative of each of the following composite functions* using the Chain Rule:

7. $F(x) = (5x^6 + 2x^3)^4$

Answer

8. $F(x) = \left(1 + x + x^2\right)^{99}$
9. $f(x) = \sqrt{5x+1}$
10. $f(x) = \dfrac{1}{\sqrt[3]{x^2 - 1}}$
11. $f(\theta) = \cos(\theta^2)$

Answer

12. $g(\theta) = \cos^2 \theta$
13. $y = x^2 e^{-3x}$

Answer

14. $f(t) = t \sin \pi t$

15. $f(t) = e^{at} \sin bt$

Answer

16. $g(x) = e^{x^2 - x}$

17. $f(x) = (2x - 3)^4 (x^2 + x + 1)^5$

Answer

18. $g(x) = (x^2 + 1)^3 (x^2 + 2)^6$

19. $h(t) = (t + 1)^{2/3} (2t^2 - 1)^3$

Answer

20. $F(t) = (3t - 1)^4 (2t + 1)^{-3}$

21. $y = \sqrt{\dfrac{x}{x + 1}}$

22. $y = \left(x + \dfrac{1}{x}\right)^5$

23. $y = e^{\tan \theta}$

Answer

24. $f(t) = 2^{t^3}$

25. $g(u) = \left(\dfrac{u^3 - 1}{u^3 + 1}\right)^8$

Answer

26. $s(t) = \sqrt{\dfrac{1 + \sin t}{1 + \cos t}}$

27. $r(t) = 10^{2\sqrt{t}}$

Answer

28. $f(z) = e^{z/(z-1)}$

29. $H(r) = \dfrac{(r^2 - 1)^3}{(2r + 1)^5}$

30. $J(\theta) = \tan^2(n\theta)$

31. $F(t) = e^{t \sin 2t}$

32. $F(t) = \dfrac{t^2}{\sqrt{t^3 + 1}}$

33. $G(x) = 4^{C/x}$

34. $U(y) = \left(\dfrac{y^4 + 1}{y^2 + 1}\right)^5$

35. $y = \cos\left(\dfrac{1 - e^{2x}}{1 + e^{2x}}\right)$

Answer

36. $y = x^2 e^{-1/x}$

37. $y = \cot^2(\sin \theta)$

55.

a. Find an equation of the tangent line to the curve $y = 2/(1 + e^{-x})$ at the point $(0, 1)$.

b. Illustrate part (a) by graphing the curve and the tangent line on the same screen.

56.

a. The curve $y = |x|/\sqrt{2 - x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point $(1, 1)$.

59. Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

Answer

60. At what point on the curve $y = \sqrt{1+2x}$ is the tangent line perpendicular to the line $6x + 2y = 1$?

61. If $F(x) = f(g(x))$, where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

Answer

62. If $h(x) = \sqrt{4+3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.

63. A table of values for $f$, $g$, $f'$, and $g'$ is given.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 6 |
| 2 | 1 | 8 | 5 | 7 |
| 3 | 7 | 2 | 7 | 9 |

a. If $h(x) = f(g(x))$, find $h'(1)$.

Answer

b. If $H(x) = g(f(x))$, find $H'(1)$.

**II** For each of the following curves, find all *critical points* (i.e., points for which dy/dx = 0).

1. $y = (x+1)^5(2x-1)^8$
2. $y = e^{-3x}(x+4)^9$
3. $y = \dfrac{(3x-5)^5}{(2x+1)^3}$
4. $y = x + \sin x$
5. $y = 13x + 3\sin 4x$


*It is often better to be in chains than to be free.*

- Franz Kafka, **The Trial**