# DISCUSSION QUESTIONS: 14 - 21 OCTOBER 2019

## IMPLICIT AND LOGARITHMIC DIFFERENTIATION

**I** Let $G(x) = (2x - 9)^{44}(3x + 4)^{15}$. Find all the critical points of G. Classify the critical points using the first derivative test. Sketch.

**II** For each of the following curves, find all *critical points* (i.e., points for which dy/dx = 0).

1. $y = (x+1)^5(2x-1)^8$
2. $y = (x+1)^5 e^{3x}$
3. $y = \dfrac{(3x-5)^5}{(2x+1)^3}$
4. $y = x + \sin x$
5. $y = 13x + 3\sin 4x$

**III** 1. Let $g(x) = x^5 e^{3x}$. Find all the critical points of g. Classify the critical points using the first derivative test. Sketch.

2. Using *implicit differentiation*, find dy/dx for each of the following implicitly defined curves:

(a) $xy + x + y = y \sin x$

(b) $\tan x + \sec y = x + y + 2019$

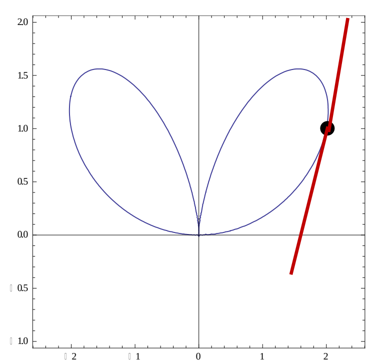
(c) $xy^4 - \tan x = e^y + 1234$

3. Find $d^2y/dx^2$ if $xy - 2x = y \sin x$

4. Find an equation of the tangent line to the *bifolium*

$$4x^4 + 8x^2y^2 - 25x^2y + 4y^4 = 0$$

at the point P = (2, 1).



5. Using implicit differentiation, find dy/dx for each of the following inverse trig functions.

y = arcsin x, y = arctan x, and y = arcsec x.

6. Differentiate each of the following functions:

(a) $y = \arcsin(3x)$

(b) $y = \arccos(5x - 13)$

(c) $y = (\text{arcsec } x) / x$

(d) $y = \arctan x + 3 \arcsin x$

(e) $y = \arctan( (x - 1)/(x + 1))$

7. Let $y = u^3 + 1$ and $u = 5 \arcsin x$. Compute dy/dx

8. Let $z = \arctan u$ and $u = e^x$. Compute dz/dx.

9. (a) Can you find a formula for d/dx (f (x) g(x) h(x))? (Called *Leibniz rule*.)

(b) Can you extend this result to a product rule for four or more factors?

(c) Using your result from (b), compute d/dx $\{5(x^3) (\cos x) (\ln x) e^x \}$

(d) Find any and all critical points of the function: $y = (x^2 + 3) (x - 5) e^x$

10. Using *logarithmic differentiation*, find dy/dx if:

(a) $y = \dfrac{x(x-9)^5\sqrt{x+5}}{x^5+99}$

(b) $y = 7(x-9)^3(x^3+x+1)^5$

(c) $y = \left(\sin^3 x\right)\left(\tan^5 x\right)(\ln x)^2$

(d) $y^x = (x+1)^{3x}$

(e) $y^{\sin x} = (\ln x)^y$

11. (a) Let $y = (\arctan t)^7$. Compute dy/dt.

(b) Let $g(x) = \cos(\ln x)$ Compute $g^{(100)}(x)$ and $g^{(101)}(x)$.

(c) Let $x = (\sinh (4t))^{1/2}$. Compute dx/dt.

(d) Let $z = (\ln(a + bx))^c$, where *a*, *b*, and *c* are constants. Compute dz/dx.

12. Given the implicit curve $y^2 = \cos(xy) - 3x$, find $\frac{dy}{dx}$.

(b) Find equations of the tangent and normal lines to the curve

$$(y - x)^2 = 2x + 4 \text{ at the point } P = (6, 2).$$

13.

[6 points] A curve $\mathcal{C}$ gives $y$ as an implicit function of $x$. This curve passes through the point $(-2, 1)$ and satisfies

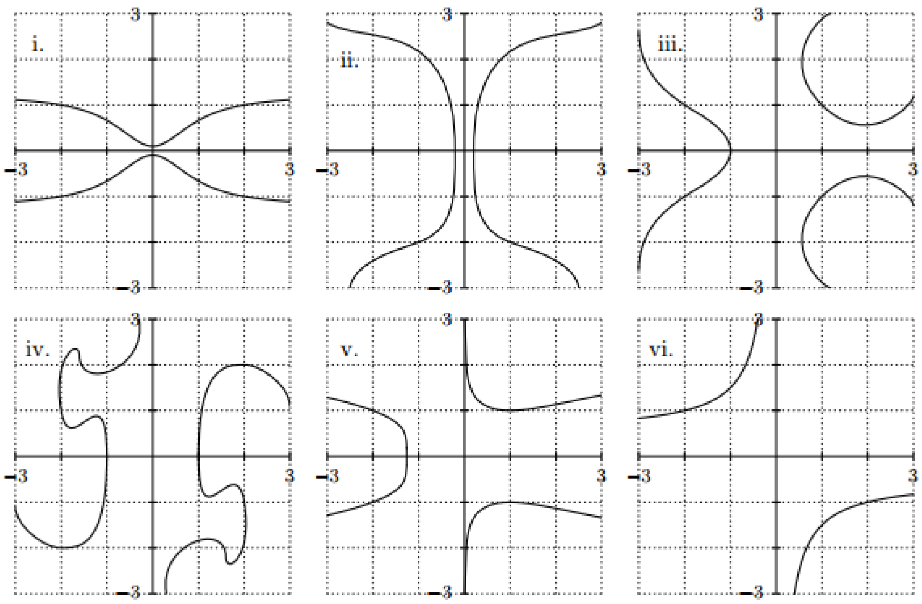
$$\frac{dy}{dx} = \frac{x^2 - y^4}{2xy^3}.$$

**a.** [1 point] One of the values below is the slope of the curve $\mathcal{C}$ at the point $(-2, 1)$. Circle that one value.

**Answer:** The slope at $(-2, 1)$ is

$-\frac{3}{16}$  $-\frac{1}{4}$  $-\frac{3}{8}$  $-\frac{1}{2}$  $-\frac{5}{8}$  $-\frac{3}{4}$  $-\frac{15}{16}$

**b.** [5 points] One of the following graphs is the graph of the curve $\mathcal{C}$. Which of the graphs i-vi is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.




**IV** 1. Given $y = \tan^2(\pi u/8)$ and $u = 1 + 2x^2 - 4x^3 + 3$, find dy/dx when x = 1.

2. Sketch the curve $y = (2x - 1)^4(3x + 1)^5$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

3. Sketch the curve $y = e^x(x - 1)^4$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

4. Show that the derivative of ln x is 1/x. (*Hint:* Let y = ln x; then $x = e^y$.)

5. Find dy/dx if y = ln(sec x + tan x) and simplify your answer.

6. Find dx/dt if x(t) = ln(ln(t)).

**V** Using implicit differentiation, find dy/dx:

1. $y + x = xy + 7$

2. $y^2 = x^2 + \sin xy$

3. $y \sin \frac{1}{y} = 1 - xy$

**VI** 1. Prove the power rule for *rational* exponents, *viz.*

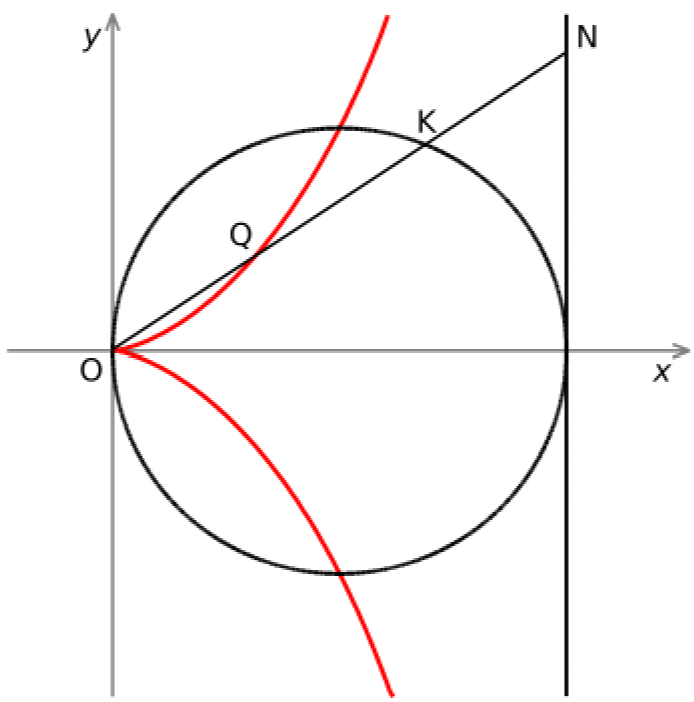
$$(d/dx)\, x^p = px^{p-1} \text{ if } p \text{ is rational.}$$

2. Find $d^2y/dx^2$ if $y^2 + xy = 1$.

3. Consider the curve defined implicitly by: $x^2 + xy - y^2 = 1$. Verify that the point P = (2, 3) lies on this curve. Find the equations of the *tangent* and *normal* lines to this curve at the point *P*.

4. Find equations for the *tangent* and *normal* lines to the *cissoid of Diocles* (from 200 B.C.):

$$y^2(2 - x) = x^3 \text{ at } Q = (1, 1).$$




**VII** Find dy/dx for each of the following:

1. $y = \arcsin(2x + 5)$

2. $y = \arctan\left(\frac{1}{x}\right)$

3. $y = \ln(arc\sec x)$

4. $y = \left(\arcsin(x^2)\right)^5$

**VIII** Using logarithmic differentiation, find dy/dx for each of the following:

1. $y = x(x+1)^5(3x-4)^{11}$

2. $y = \dfrac{5x+7}{\sqrt{3x+5}}$

3. $y = \sqrt{\dfrac{x(3x+1)(2x+5)}{(x-4)(7x-1)}}$

*To most outsiders, modern mathematics is unknown territory. Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts. Few realize that the world of modern mathematics is rich with vivid images and provocative ideas.*

**- Ivars Peterson**