# MATH 161 CLASS DISCUSSION

## 2 OCTOBER 2019

### ➢ The Game of Antiderivatives ("method of judicious guessing")

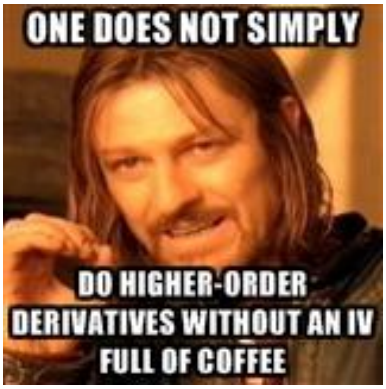
Find an antiderivative for each of the following functions

(a) $\cos x$ (b) $2019$ (c) $6x^5 + 5x^4 + e^5$ (d) $\sec^2 x$ (e) $x^9 + 7x^3 + 1$

(f) $5e^x + \sec x \tan x$ (g) $19 \sin x$ (h) $x - \cos x + 5$

(i) $(2 + 3x)^2$ (j) $\frac{3x^3+4x^2+9x}{x}$ (k) $\tan^2 x$ (Hint: Use a familiar trig identity.)

## HIGHER-ORDER DERIVATIVES




1. Verify that the power rule also works for negative exponents, by using the quotient rule.
2. Explain why $\frac{d}{dx}\, b^x = k\, b^x$
3. Find the first *three* derivatives of each of the following functions.

   $(A)\quad y = ax^2 + bx + c$

   $(B)\quad y = 2x^3 + \frac{1}{x^2} + e^x$

   *(Here use the shortcut for differentiating $x^n$ for negative values of n.)*

   $(C)\quad y = xe^x$

   $(D)\quad y = \sin x$

   $(E)\quad y = x \sin x$

   $(F)\quad y = x^{101}$

4. (a) If $\frac{d}{dx} e^{4x} = 4e^{4x}$, find $\frac{d^{99}}{dx^{99}} e^{4x}$

(b) If (d/dx) sin 5x = 5 cos 5x, and $\frac{d}{dx}$ cos 5x = - 5 sin 5x, find $\frac{d^{2019}}{dx^{2019}}$ sin 5x.

5. If $f(x) = x^{1/2}$, find $f^{(4)}(x)$. *(Here assume that the shortcut for differentiating $x^n$ is valid for rational n.)*

*6. . Discuss concavity and the second-derivative test for local extrema.*


7. If $x(t) = 3t^3 - 4t + 1$ is the position (measured in meters) of Charlotte on the x-axis at time *t* (measured in hours), find Charlotte's *velocity* and *acceleration* at time t = 2 hrs.

8. If $F(x) = x^m$, find $F^{(m)}(x)$. (Assume that *m* is a positive integer.)

9. Let y = ln x. Given that $\frac{dy}{dx} = \frac{1}{x}$ find $d^4y/dx^4$. Can you find $d^{10}y/dx^{10}$ ?

10. *(University of Michigan)* Consider the following table giving values, rounded to three decimal places, of a function *f*(*x*).

| *x* | 0 | 0.5 | 1 |
|---|---|---|---|
| *f*(*x*) | 0 | 0.247 | 0.841 |

(a) Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

(b) Estimate $f(1.25)$ being sure your work is clear.

(c) Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

(d) Based on your work in (a) and (c), is your estimate in (b) an over- or underestimate? Explain.

*11. (University of Michigan)* A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Mertz hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time *t* = 0 and later times is shown in the table of data below (in which *t* is in seconds and the velocities are in m/s).

| **between *t* = 0 seconds and *t* =** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **the average velocity is** | **−5** | **−10** | **−11.67** | **−9** | **−7.2** |

a) Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $h(t)$ | 40 | _____ | _____ | _____ | _____ | _____ |

b) Based on your work from (a), is $h''(1) > 0$, $< 0$, or $= 0$? Is $h''(3) > 0$, $< 0$, or $= 0$? Explain.

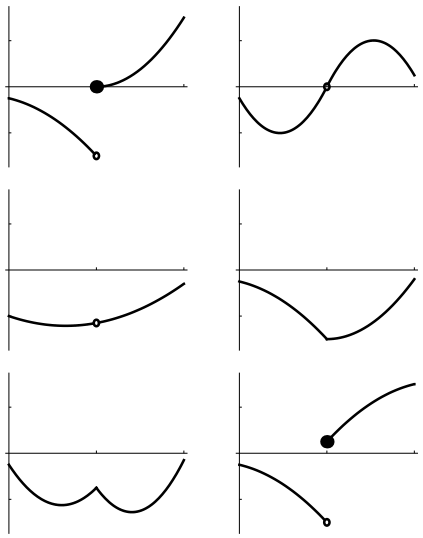
***9.*** For each of the descriptions of a function $f$ that follow, indicate which of the graphs match the description. For each description there may be no, one, or several graphs that match; write **none** if no graphs match the description. You may need to use a graph more than once. In each case, you should assume that $f$ is defined only on the domain $[0, 2]$.

**a.** $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is continuous everywhere except at $x = 1$.

**b.** $f''(x) > 0$ for all $x < 1$; $f''(x) < 0$ for all $x > 1$ ; and $f(x)$ is differentiable everywhere except at $x = 1$.

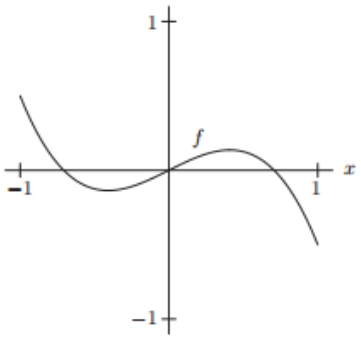
**c.** $f''(x) < 0$ for all $x < 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x) < 0$ for all $x = 1$.

**d.** $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.



***10.*** The graph of a function $f$ is given in the Figure below. If $f$ is a polynomial of degree 3, then the value of $f'''(0)$ *is*

(a) Positive (b) Negative (c) Zero




***11.*** The graph of a function in Figure 3.5. If $f$ is a polynomial of degree 3, then the value of $f'(0), f'(0), and\ f'''(0)\ are\ (respectively)$

(a) $-,+,+$ (b) $-,-,-$ (c) $-,+,-$
(d) $-,+,+$ (e) $+,-,+$ (f) $+,+,+$

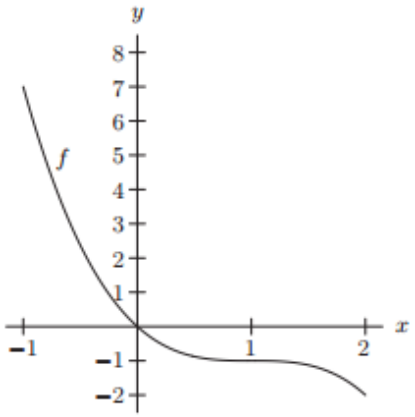
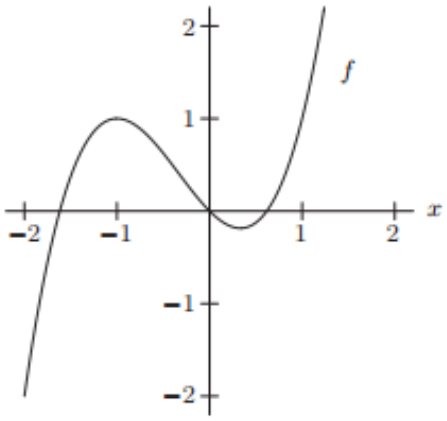
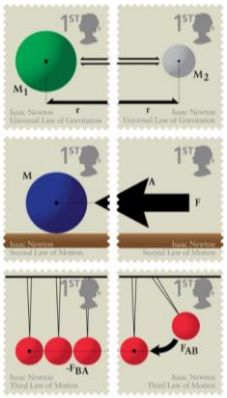



Figure 3.5

**13.** The graph of a function $f$ is given below. If f is a polynomial of degree 3, then the values of $f'(0), f''(0), and\ f'''(0) are\ (respectively)$

(a) $-,-,+$ (b) $-,0,-$ (c) $-,+,-$
(d) $-,+,+$ (e) $+,-,+$ (f) $+,+,+$




1ST
M1
M2
r
M
F
-FBA
FAB