

DISCUSSION QUESTIONS: 14 - 21 OCTOBER 2019

IMPLICIT AND LOGARITHMIC DIFFERENTIATION

I Let $G(x) = (2x - 9)^{44}(3x + 4)^{15}$. Find all the critical points of G . Classify the critical points using the first derivative test. Sketch.

II For each of the following curves, find all *critical points* (i.e., points for which $dy/dx = 0$).

1. $y = (x + 1)^5(2x - 1)^8$

2. $y = (x+1)^5 e^{3x}$

3. $y = \frac{(3x - 5)^5}{(2x + 1)^3}$

4. $y = x + \sin x$

5. $y = 13x + 3 \sin 4x$

III 1. Let $g(x) = x^5 e^{3x}$. Find all the critical points of g . Classify the critical points using the first derivative test. Sketch.

2. Using *implicit differentiation*, find dy/dx for each of the following implicitly defined curves:

(a) $xy + x + y = y \sin x$

(b) $\tan x + \sec y = x + y + 2019$

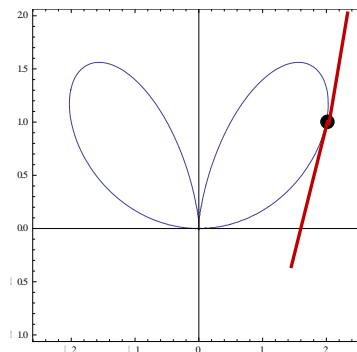
(c) $xy^4 - \tan x = e^y + 1234$

3. Find d^2y/dx^2 if $xy - 2x = y \sin x$

4. Find an equation of the tangent line to the *bifolium*

$$4x^4 + 8x^2y^2 - 25x^2y + 4y^4 = 0$$

at the point $P = (2, 1)$.



5. Using implicit differentiation, find dy/dx for each of the following inverse trig functions.

$y = \arcsin x$, $y = \arctan x$, and $y = \operatorname{arcsec} x$.

6. Differentiate each of the following functions:

(a) $y = \arcsin(3x)$

(b) $y = \arccos(5x - 13)$

(c) $y = (\operatorname{arcsec} x) / x$

(d) $y = \arctan x + 3 \arcsin x$

(e) $y = \arctan((x - 1)/(x + 1))$

7. Let $y = u^3 + 1$ and $u = 5 \arcsin x$. Compute dy/dx

8. Let $z = \arctan u$ and $u = e^x$. Compute dz/dx .

9. (a) Can you find a formula for $d/dx (f(x) g(x) h(x))$? (Called *Leibniz rule*.)

(b) Can you extend this result to a product rule for four or more factors?

(c) Using your result from (b), compute $d/dx \{5(x^3) (\cos x) (\ln x) e^x\}$

(d) Find any and all critical points of the function: $y = (x^2 + 3)(x - 5) e^x$

10. Using *logarithmic differentiation*, find dy/dx if:

(a) $y = \frac{x(x - 9)^5 \sqrt{x + 5}}{x^5 + 99}$

(b) $y = 7(x - 9)^3 (x^3 + x + 1)^5$

(c) $y = (\sin^3 x)(\tan^5 x)(\ln x)^2$

(d) $y^x = (x + 1)^{3x}$

(e) $y^{\sin x} = (\ln x)^y$

11. (a) Let $y = (\arctan t)^7$. Compute dy/dt .

(b) Let $g(x) = \cos(\ln x)$. Compute $g^{(100)}(x)$ and $g^{(101)}(x)$.

(c) Let $x = (\sinh(4t))^{1/2}$. Compute dx/dt .

(d) Let $z = (\ln(a + bx))^c$, where a , b , and c are constants. Compute dz/dx .

12. Given the implicit curve $y^2 = \cos(xy) - 3x$, find $\frac{dy}{dx}$.

(b) Find equations of the tangent and normal lines to the curve

$$(y - x)^2 = 2x + 4 \text{ at the point } P = (6, 2).$$

13.

[6 points] A curve C gives y as an implicit function of x . This curve passes through the point $(-2, 1)$ and satisfies

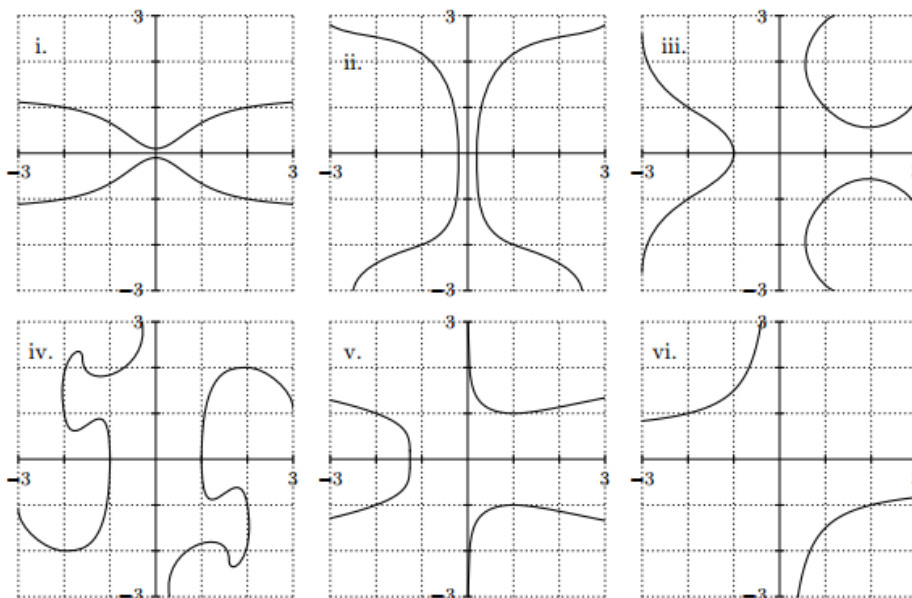
$$\frac{dy}{dx} = \frac{x^2 - y^4}{2xy^3}.$$

- a. [1 point] One of the values below is the slope of the curve C at the point $(-2, 1)$. Circle that one value.

Answer: The slope at $(-2, 1)$ is

$$-\frac{3}{16} \quad -\frac{1}{4} \quad -\frac{3}{8} \quad -\frac{1}{2} \quad -\frac{5}{8} \quad -\frac{3}{4} \quad -\frac{15}{16}$$

- b. [5 points] One of the following graphs is the graph of the curve C . Which of the graphs i-vi is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.



IV 1. Given $y = \tan^2(\pi u/8)$ and $u = 1 + 2x^2 - 4x^3 + 3$, find dy/dx when $x = 1$.

2. Sketch the curve $y = (2x - 1)^4(3x + 1)^5$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

3. Sketch the curve $y = e^x(x - 1)^4$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.

4. Show that the derivative of $\ln x$ is $1/x$. (*Hint:* Let $y = \ln x$; then $x = e^y$.)

5. Find dy/dx if $y = \ln(\sec x + \tan x)$ and simplify your answer.

6. Find dx/dt if $x(t) = \ln(\ln(t))$.

V Using implicit differentiation, find dy/dx :

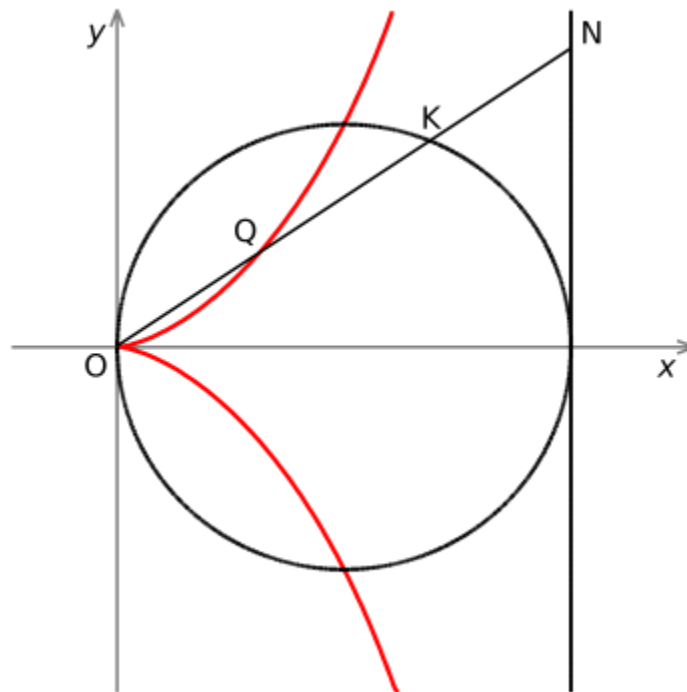
1. $y + x = xy + 7$
2. $y^2 = x^2 + \sin xy$
3. $y \sin \frac{1}{y} = 1 - xy$

VI 1. Prove the power rule for *rational* exponents, viz.

$$(d/dx) x^p = px^{p-1} \text{ if } p \text{ is rational.}$$

2. Find d^2y/dx^2 if $y^2 + xy = 1$.
3. Consider the curve defined implicitly by: $x^2 + xy - y^2 = 1$. Verify that the point $P = (2, 3)$ lies on this curve. Find the equations of the *tangent* and *normal* lines to this curve at the point P .
4. Find equations for the *tangent* and *normal* lines to the *cisoid of Diocles* (from 200 B.C.):

$$y^2(2 - x) = x^3 \text{ at } Q = (1, 1).$$



VII Find dy/dx for each of the following:

1. $y = \arcsin(2x + 5)$
2. $y = \arctan\left(\frac{1}{x}\right)$
3. $y = \ln(\text{arc sec } x)$
4. $y = (\arcsin(x^2))^5$

VIII Using logarithmic differentiation, find dy/dx for each of the following:

1. $y = x(x + 1)^5(3x - 4)^{11}$

2. $y = \frac{5x + 7}{\sqrt{3x + 5}}$

3. $y = \sqrt{\frac{x(3x + 1)(2x + 5)}{(x - 4)(7x - 1)}}$

To most outsiders, modern mathematics is unknown territory. Its borders are protected by dense thickets of technical terms; its landscapes are a mass of indecipherable equations and incomprehensible concepts. Few realize that the world of modern mathematics is rich with vivid images and provocative ideas.

- **Ivars Peterson**