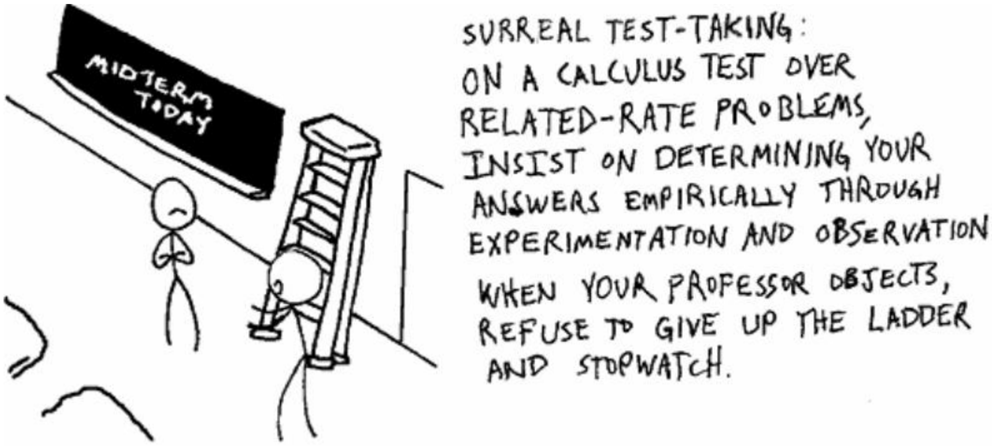
DISCUSSION QUESTIONS: 28 OCTOBER 2019

RELATED RATES, CONTINUED



1. (a) Let $y = u^3 + 1$ and $u = 5 \arcsin x$. Compute dy/dx

(b) Let $z = \arctan u$ and $u = e^x$. Compute dz/dx.

2. Prove the power rule for *rational* exponents, *viz.* $(d/dx)\, x^p = p\, x^{p-1}$ if p is rational.

3. A cone-shaped coffee filter of radius 6 cm and depth 10 cm contains water, which drips out through a hole at the bottom at a constant rate of 1.5 cm^3 per second.

(a) If the filter starts out full, how long does it take to empty?

(b) Find the volume of water in the filter when the depth of the water is h cm.

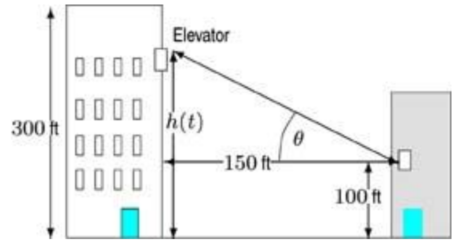
(c) How fast is the water level falling when the depth is 8 cm?

4. A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling toward the gas station from the east, chasing a stolen truck that is traveling north away from the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles from the gas station going 80 mph. At this moment:

(a) Is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car and the truck.)

(b) How does your answer change if the truck is traveling at a speed of 70 mph instead of 80 mph?

5. For the amusement of the guests, some hotels have elevators on the outside of the building. One such hotel is 300 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 30 ft/sec, starting at time $t = 0$, where t is time in seconds. Let □ be the angle between the line of your horizon and your line of sight to the elevator. (See the figure below)



(a) Find a formula for h(t), the elevator's height above the ground as it descends from the top of the hotel.

(b) Using your answer to part (a), express □ as a function of time *t* and find the rate of change of θ with respect to *t*.

(c) The rate of change of □ is a measure of how fast the elevator appears to you to be moving. At what height is the elevator when it appears to be moving fastest?

6.

The area of a triangle with sides of lengths a and b and contained angle θ is

$$A = \frac{1}{2}ab\sin\theta$$

a. If $a = 2$ cm, $b = 3$ cm, and θ increases at a rate of 0.2 rad/min, how fast is the area increasing when $\theta = \pi/3$?

b. If $a = 2$ cm, b increases at a rate of 1.5 cm/min, and θ increases at a rate of 0.2 rad/min, how fast is the area increasing when $b = 3$ cm and $\theta = \pi/3$?

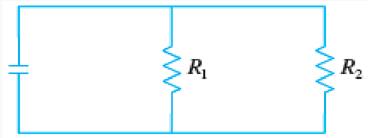
c. If a increases at a rate of 2.5 cm/min, b increases at a rate of 1.5 cm/min, and θ increases at a rate of 0.2 rad/min, how fast is the area increasing when $a = 2$ cm, $b = 3$ cm, and $\theta = \pi/3$?

7.

If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

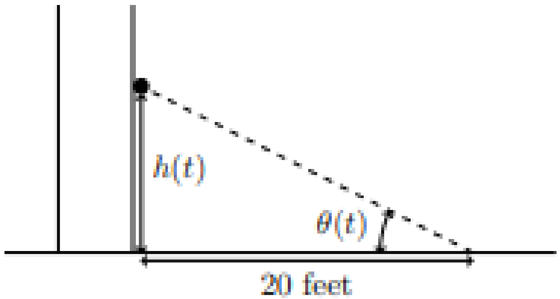
If R_1 and R_2 are increasing at rates of $0.3\ \Omega/s$ and $0.2\ \Omega/s$, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



(University of Michigan problems)

M 1. Walking through the Loyola campus one autumn day, Albertine sees a squirrel running down the trunk of a tree. The trunk of the tree is perfectly straight and makes a right angle with the ground. She stops 20 feet away from the tree and lies down on the ground to watch the squirrel.

Suppose h(t) is the distance in feet between the squirrel and the ground, and θ(t) is the angle in radians between the ground and Albertine's line of sight to the squirrel, with *t* being the amount of time in seconds since she stopped to watch the squirrel.



(a) Write an equation relating h(t) and θ(t). (Hint: Use the tangent function.)

(b) If θ(t) is decreasing at 1/5 of a radian per second when θ(t) = π/3, how fast is the squirrel moving at that time?

(c) For the last second, before the squirrel reaches the ground, it is moving at a constant speed of 20 feet per second. Suppose θ′ (t) = −3/4 at some point during this last second. How high is the squirrel at this time?

M 2. Suppose that a ring made entirely of gold and platinum is made from *g* ounces of gold and *p* ounces of platinum and that gold costs *h* dollars per ounce and platinum costs *k* dollars per ounce. Then the *value*, in dollars, of the ring is given by **v = gh + pk**.

(a) Swann has a ring made entirely of gold and platinum. Swann's ring is made from 0.25 ounces of gold and 0.15 ounces of platinum. Suppose that the cost of gold is *decreasing* at an instantaneous rate of $20 per ounce per year, while the cost of platinum is *increasing* at an instantaneous rate of $30 per ounce per year. At what instantaneous rate is the value of Swann's ring increasing or decreasing? Remember to include units in your answer.

(b) Odette wants to design a ring made entirely of gold and platinum with a current value of $900. Currently, gold costs $1200 per ounce, and platinum costs $1500 per ounce. Let w(p) be the total weight of Odette's ring, in ounces, if *p* ounces of platinum are used.

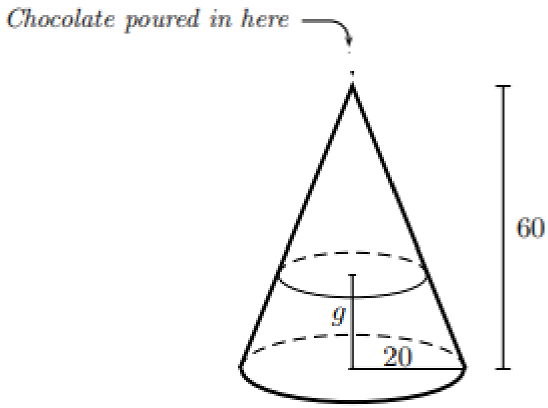
(i) In the context of this problem, what is the domain of w(p)?

(ii) Find a formula for w(p). No variables other than *p* should appear in your answer.

(iii) How much gold and platinum should be in the ring if Odette wants to *minimize* the weight of the ring?

(iv)

M3 A wicked villain decides to relax with handmade chocolate before he heads to his farmhouse. The process of making the chocolate involves pouring molten chocolate into a mold. The mold is a cone with height 60 mm and base radius 20 mm. Freddy places the mold on the ground and begins pouring the chocolate through the apex of the cone. A diagram of the situation is shown below.



In case they are helpful, recall the following formulas for a cone of radius r and height h:

$$\text{Volume} = \frac{1}{3}\pi r^2 h \quad \text{and} \quad \text{Surface Area} = \pi r(r + \sqrt{h^2 + r^2}).$$

(a) Let g be the depth of the chocolate, in mm, as shown in the diagram above. What is the value of g when Freddy has poured 20,000 mm^3 of chocolate into the mold? Show your work carefully, and make sure your answer is accurate to at least two decimal places.

(b) How fast is the depth of the chocolate in the mold (g in the diagram above) changing when Freddy has already poured 20,000 mm^3 of chocolate into the mold if he is pouring at a rate of 5,000 mm^3 per second at this time? Show your work carefully and make sure your answer is accurate to at least two decimal places. Be sure to include units.