CLASS DISCUSSION: 4 OCTOBER 2019

The Game of Anti-Derivatives:

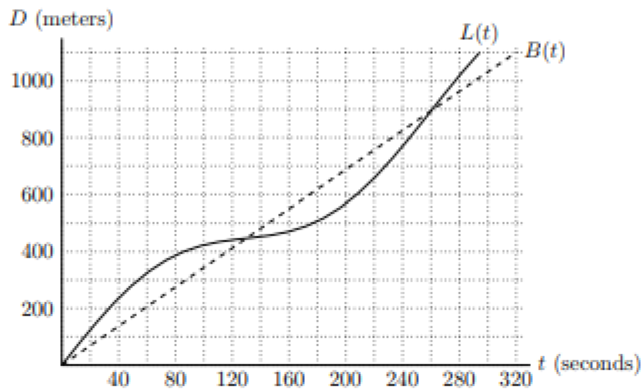
Using the method of "judicious guessing," find an anti-derivative of each of the following functions:

(a) x^{99} (b) $\frac{3}{x^2}$ (c) $\frac{1+x}{\sqrt{x}}$ (d) $x(x+7)$ (e) $(x^3 + x + 1)(x^3 - 2x)$

(f) $tan^2\, x + 17$ (g) $e^e\, e^x$ (f) $\tan x\, sec^2 x$ (g) $sin^2 x + cos^2 x$

Review:

[U. of Michigan] Puss and Boots decided to race a straight portion of Sheridan Road that is 1.1 km. long. Let L(t) and B(t) be Puss' and Boots' respective distances from their starting point *t* seconds after the race began. A graph of L(t) and B(t) is shown below:



(a) Who won the race?

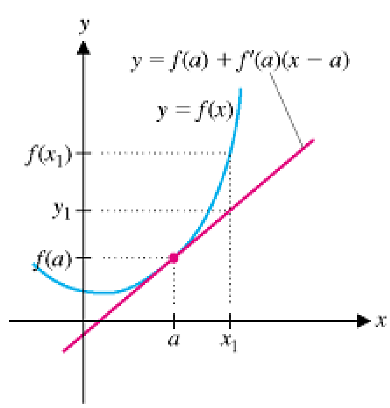
(b) Estimate the time at which Puss and Boots were running at the same speed.

(c) Estimate Puss' average velocity over the first 100 seconds of the race. Include units.

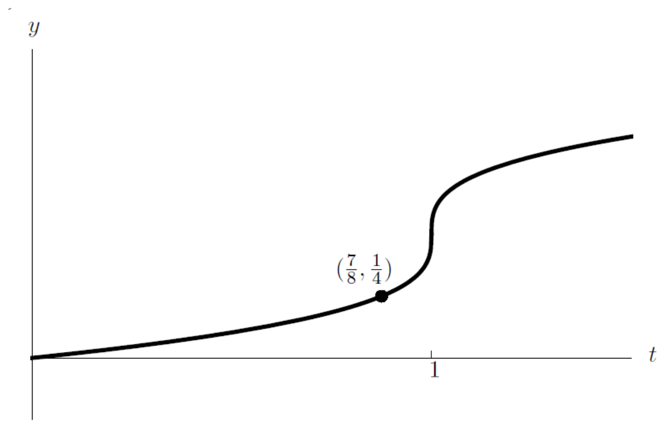
(d) Estimate Puss' instantaneous velocity 40 seconds after the race began. Include appropriate units.

(e) 160 seconds after the race began, is Puss' acceleration positive, negative, or equal to zero?

Linear approximations



1. Find the linearization of the function $f(x) = \sqrt{x+3}$ at the point x = 1 and use it to approximate $\sqrt{3.98}$ *and* $\sqrt{4.05}$. For each approximation, is it an underestimate or an overestimate? Explain. (Here you may use the power rule short cut.)
2. Find the linearization of the function f(x) = sin x at the point x = π/6.
3. Find the linearization of the function $f(x) = (1+x)^{-3}$ at the point x = 0 and use it to approximate the value of $\frac{1}{1.003^3}$. Is your approximation an *underestimate*, or an *overestimate*? Explain.
4. *(U. Michigan)* Given below is the graph of a function h(t). Suppose j(t) is the local linearization of h(t) at t = 7/8.



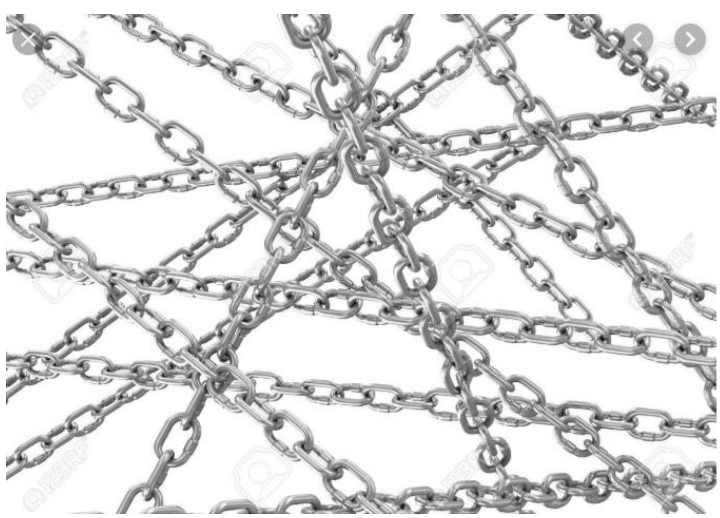
(a) Given that $h'\left(\frac{7}{8}\right) = \frac{2}{3}$, *find an expression* for *j*(t).

(b) Use your answer from (a) to approximate h(1).

(c) Is the approximation from (b) an over- or under-estimate? Explain.

(d) Using j(t) to estimate values of h(t), will the estimate be more accurate at t=1 or t = ¾? Explain.

Introducing the Chain Rule



1. For each of the following functions, determine the "inner" and the "outer" functions. If there are several options, try to choose the best.
   (a) $H(x) = \cos(e^x)$
   (b) $v(x) = e^{\cos x}$
   (c) $w(x) = \ln(\tan x + 1)$
   (d) $z(x) = 3^{4x+1}$
   (e) $h(x) = \sqrt{x\cos x + 11}$
   (f) $r(x) = (x\cos x + 11)^{55}$
   (g) $s(t) = \sqrt{\frac{1+5x}{3-x^3}}$

2. Using the chain rule, differentiate each of the following functions. First, identify the inner and outer functions.

(a) $y = (2x + 1)^3$

(b) $y = e^{4x}$

(c) $y = \tan(3x + 9)$

(d) $y = \sec(x^2 + 2019)$

History of the chain rule *[Wikepedia]***:** The chain rule seems to have first been used by Gottfried Wilhelm Leibniz. He used it to calculate the derivative of

$$\sqrt{a + bx + cx^2}$$

the composite of the square root function and the function $a + bx + cx^2$. He first mentioned it in a 1676 memoir (with a sign error in the calculation). The common notation of the chain rule is due to Leibniz. Guillaume de l'Hôpital used the chain rule implicitly in his *Analyse des infiniment petits*. The chain rule does not appear in any of Leonhard Euler's analysis books, even though they were written over a hundred years after Leibniz's discovery.