MATH 161 CLASS DISCUSSION: 16 SEPT 2019

FIRST LOOK AT THE DERIVATIVE

Introducing the derivative

1. Which of the following graphs (a) – (d) could represent the slope at every point of the function graphed below, labeled Figure 2.6?

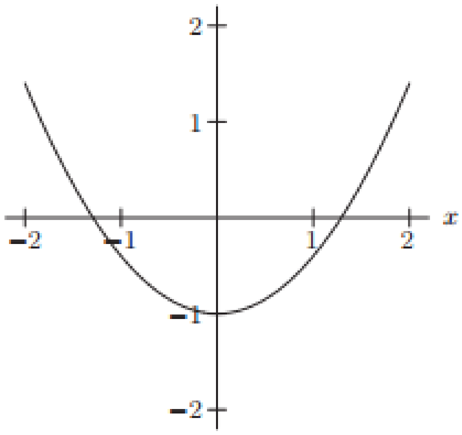
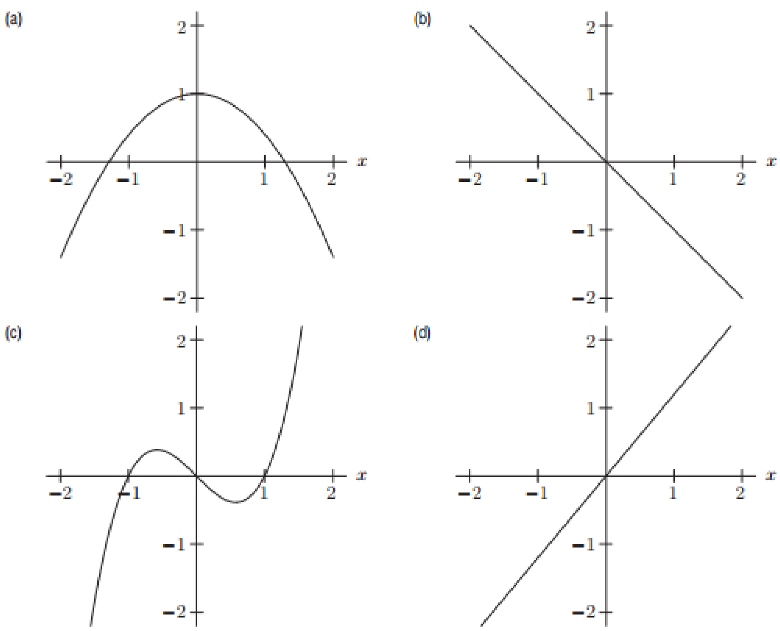


Figure 2.6



2. Which of the following graphs (a) – (d) could represent the slope at every point of the function graphed in Figure 2.11?

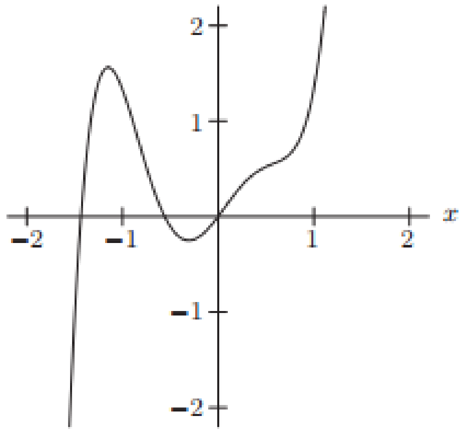


Figure 2.11

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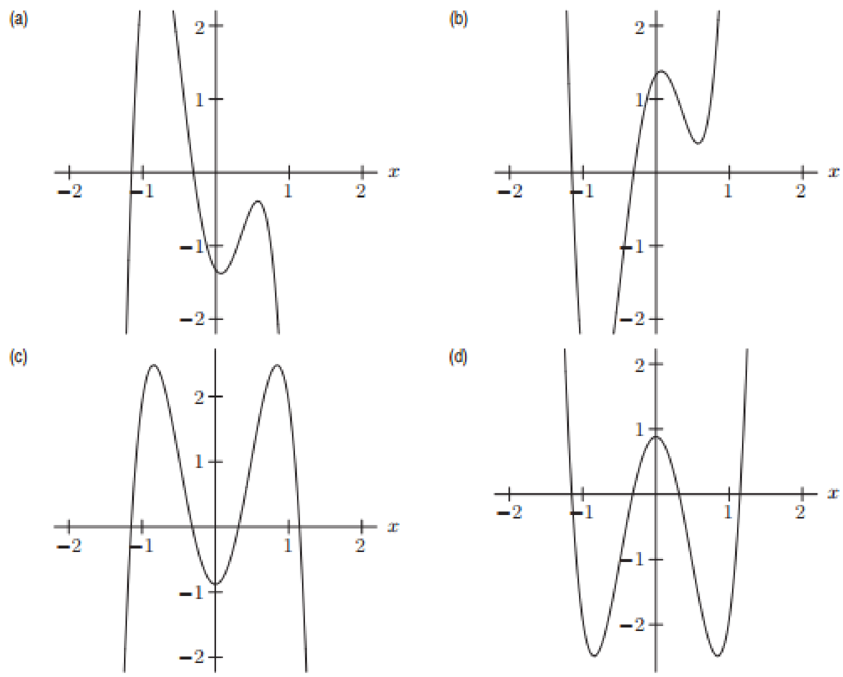
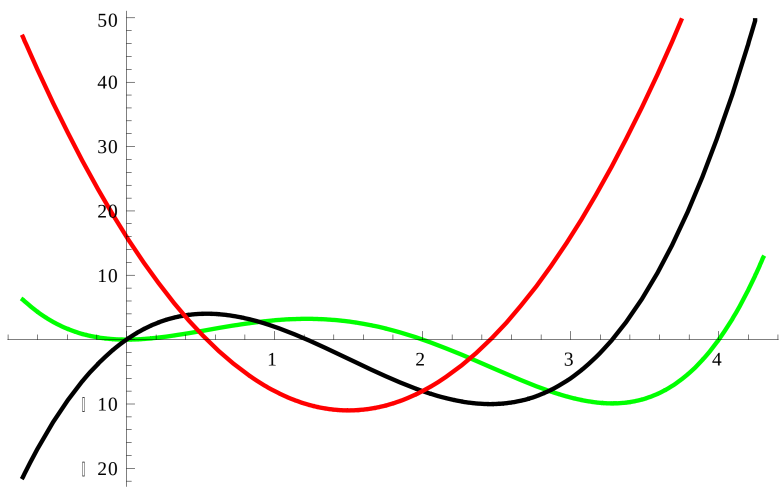
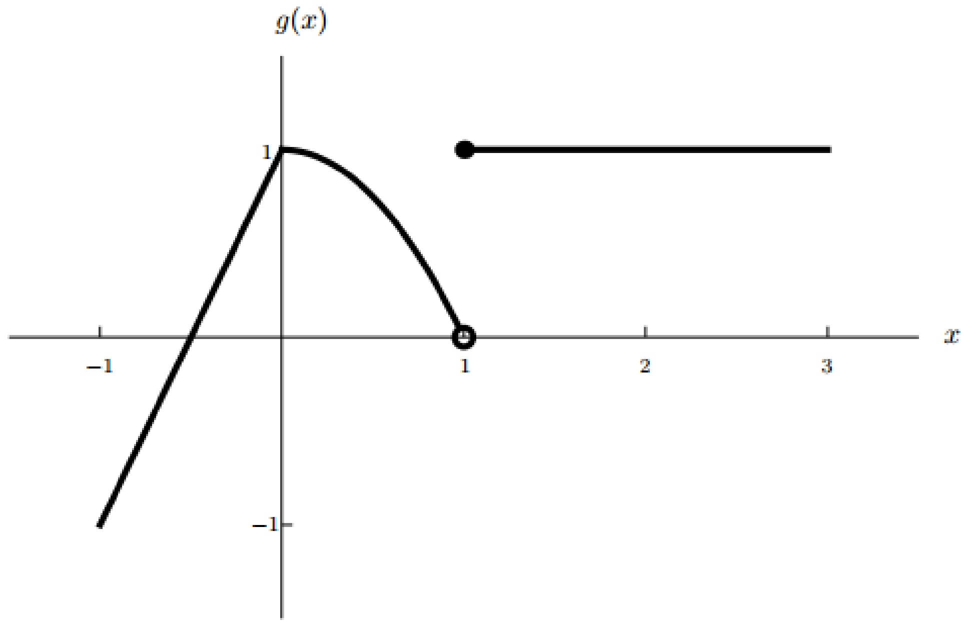


3. Mehitable, the cat, lives on the x-axis. Graphs of her *position*, *velocity*, and *acceleration* during the time interval

$-0.7 < t < 4.3$ appear below. Which is which? Explain.



4. The graph of a function g(x) is given below.



Sketch a graph of $g'(x)$. *Label the axes.*

5. The function C(r) is the total cost, in dollars, of paying off a car loan borrowed at an interest rate of r % per year.

What are the units of $C'(r) = \frac{dC}{dr}$?

(a) Year/\$ (b) \$/Year (c) \$/(%/Year) (d) (%/Year)/\$

What is the practical meaning of $C'(5)$?

(a) The rate of change of the total cost of the car loan is $C'(5)$.
(b) If the interest rate increases by 1%, then the total cost of the loan increases by about $C'(5)$.
(c) If the interest rate increases by 1%, then the total cost of the loan increases by about $C'(5)$ when the interest rate is 5%.
(d) If the interest rate increases by 5%, then the total cost of the loan increases by about $C'(5)$.

What is the sign of $C'(5)$?

(a) Positive
(b) Negative
(c) Not enough information

6. The temperature, *Y*, in Fahrenheit, of a cold yam placed in a hot oven is given by Y = g(t), where *t* is the time in minutes since the yam was placed into the oven.

(a) What is the sign of g′(t)? Why?

(b) What are the units of g′(20)? What is the *practical meaning* of the statement

g′(20) = 2?

7. For some painkillers, the size of the dose, *D*, given depends upon the weight of the patient, *W*. Thus, D = H(W), where *D* is in milligrams and *W* is in pounds.

(a) Interpret the statements H(140) = 120 and H′(140) = 3 in terms of this painkiller.
(b) Use the information in the statements in part (a) to estimate H(145).

8. Suppose that C(T) is the cost of heating Albertine's house, in dollars per day, when the *outside* temperature is *T* degrees Fahrenheit.

(a) What does C(19) = 8.67 mean in practical terms? (Use appropriate units.)

(b) What does C ′ (19) = - 0.55 mean in practical terms? (Use appropriate units.)

(c) If C(19) = 8.67 and C ′(19) = - 0.55, approximately what is the cost of heating Albertine's house when the outside temperature is 16 degrees Fahrenheit? (Use appropriate units.)

9. The cost *C* (in thousands of dollars) of building a house that is *x* square feet, by the function C = F(x).

(a) Explain the *meaning* of the statement: F(1600) = 140.

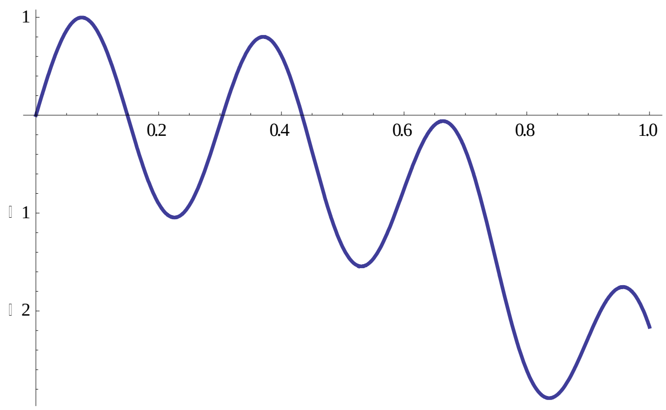
(b) Give the *practical interpretation* of the statement: F′(1600) = 0.1.

(c) Using the information given in parts (a) and (b), *estimate* the cost of building a house that is 1680 square feet.

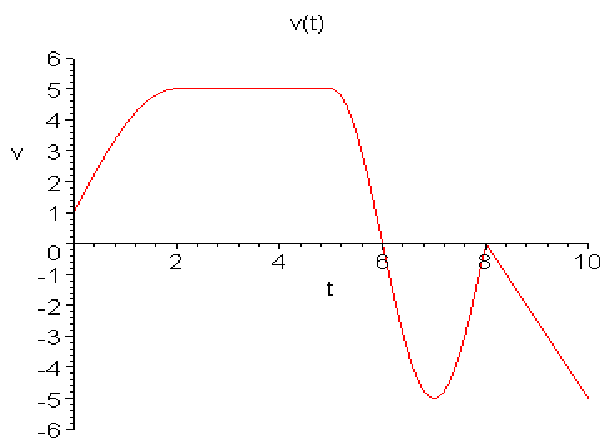
10. Use geometric differentiation on the function y = sin x. Can you guess what the derivative function is?

11. Given the following graph of y = f(x), use "geometric differentiation" to sketch the graph of dy/dx.

(If you are curious, the equation of this curve is $y = x^5 + \sin(21x) - 4x^3$)



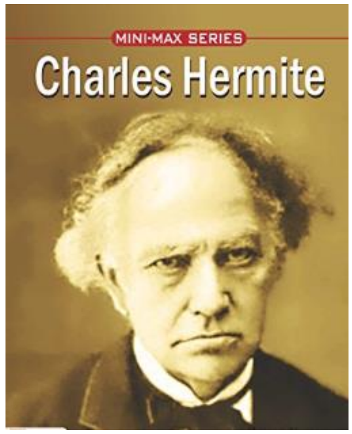
12. Below is the graph of a velocity function of Albertine riding her mountain bike. The units on the vertical axis are in *kilometers per hour*, and the units on the horizontal axis are in *hours*. Positive velocity means motion away from the starting position; negative velocity means motion toward the starting position.



Sketch a possible graph of Albertine's position function during the time interval t = 0 to t = 10.

I turn away with fright and horror from the lamentable evil of functions that do not have derivatives.

- Charles Hermite (in a letter to Thomas Jan Stieltjes)



Using Mathematica to plot derivatives

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Plot[x Sin[x], {x, 0, 8}]
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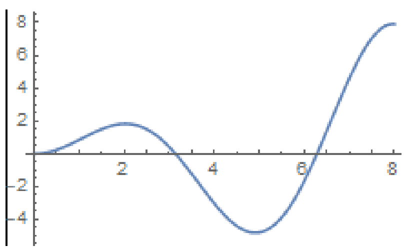


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f[x_] := x Sin[x]

f[9]

9 Sin[9]
f[9.0]

9 Sin[9]

3.70907

Plot[{f[x], f'[x]}, {x, 0, 8}]
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