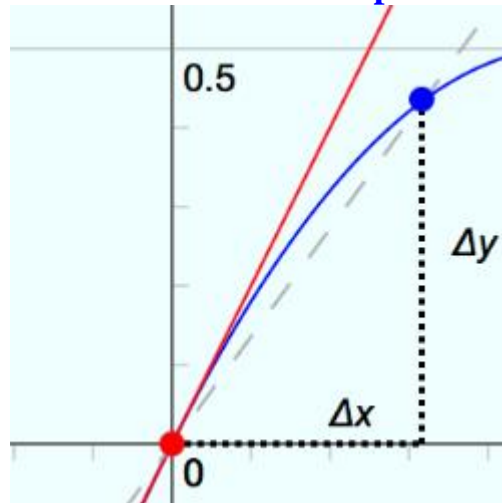


MATH 161 CLASS DISCUSSION: 18 SEPTEMBER 2019

The derivative at a point



1. Find an equation of the *tangent line* to the given curve at the given point.

(a) $y = 9x - 1789$ at $x = 5$

(b) $y = x^2 + 2019$ at $x = 2$

(c) $y = 5x^2 + 11$ at $c = 3$

(d) $y = \sqrt{x}$ at $x = 9$

(e) $y = \sin x$, at $x = 0$

(f) $y = \frac{3}{x}$ at $x = 2$

(g) $y = \frac{1}{\sqrt{x}}$ at $x = 4$

(h) $y = 1/(x - 3)^2$, $x = 4$

2. The function $C(r)$ is the total cost, in dollars, of paying off a car loan borrowed at an interest rate of r % per year.

What are the units of $C'(r) = \frac{dC}{dr}$?

(a) Year/\$

(b) \$/Year

(c) \$/(%/Year)

(d) (%/Year)/\$

What is the practical meaning of $C'(5)$?

(a) The rate of change of the total cost of the car loan is $C'(5)$.

(b) If the interest rate increases by 1%, then the total cost of the loan increases by about $C'(5)$.

(c) If the interest rate increases by 1%, then the total cost of the loan increases by about $C'(5)$ when the interest rate is 5%.

(d) If the interest rate increases by 5%, then the total cost of the loan increases by about $C'(5)$.

What is the sign of $C'(5)$?

(a) Positive

(b) Negative

(c) Not enough information

3. The temperature, Y , in Fahrenheit, of a cold yam placed in a hot oven is given by $Y = g(t)$, where t is the time in minutes since the yam was placed into the oven.

(a) What is the sign of $g'(t)$? Why?

(b) What are the units of $g'(20)$? What is the *practical meaning* of the statement

$$g'(20) = 2?$$

4. For some painkillers, the size of the dose, D , given depends upon the weight of the patient, W . Thus, $D = H(W)$, where D is in milligrams and W is in pounds.
- Interpret the statements $H(140) = 120$ and $H'(140) = 3$ in terms of this painkiller.
 - Use the information in the statements in part (a) to estimate $H(145)$.
5. Suppose that $C(T)$ is the cost of heating Albertine's house, in dollars per day, when the *outside* temperature is T degrees Fahrenheit.
- What does $C(19) = 8.67$ mean in practical terms? (Use appropriate units.)
 - What does $C'(19) = -0.55$ mean in practical terms? (Use appropriate units.)
 - If $C(19) = 8.67$ and $C'(19) = -0.55$, approximately what is the cost of heating Albertine's house when the outside temperature is 16 degrees Fahrenheit? (Use appropriate units.)
6. The cost C (in thousands of dollars) of building a house that is x square feet is given by the function $C = F(x)$.
- Explain the *meaning* of the statement: $F(1600) = 140$.
 - Give the *practical interpretation* of the statement: $F'(1600) = 0.1$.
 - Give the *practical interpretation* of the statement: $F'(1600) = 0.1$.
 - Using the information given in parts (a) and (b), *estimate* the cost of building a house that is 1680 square feet.
7. The parabola $y = x^2 + x + c$ is tangent to the line $y = 3x$. Find c . (Include a picture in your explanation.)
8. Find a parabola, $y = ax^2 + bx + c$, that passes through the point (1, 4) and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2 respectively.
9. Find equations of any (and all) tangent lines to the parabola $y = x^2 + 1$ that have *x-intercept* of $-4/3$. Sketch.
10. Archy lives on the x -axis. His position at time t (hours) is
- $$s(t) = 4t^3 - 15t^2 + 12t + 1 \text{ (cm)}.$$
- Assume that Archie was born at time $t = 0$ and that $\frac{ds}{dt} = 12t^2 - 30t + 12$
- What is Archy's *position* at time $t = 1$?
 - What is Archy's instantaneous *velocity* at time $t = 1$?
 - When is Archy moving *toward the left*? (Give one or more time intervals.)
11. The quantity, Q mg, of nicotine in the body t minutes after a cigarette is smoked is given by $Q = g(t)$.
- Using a complete sentence, interpret the statement $g(20) = 0.36$.
 - Using a complete sentence, interpret the statement $g'(20) = -0.002$.
 - Using the information that you obtained above, estimate $g(23)$.
12. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 2017$
- $$\text{You may assume that } f'(x) = 6x^2 - 6x - 12$$
- Find any and all points (only their x -coordinates) at which the tangent line to $y = f(x)$ is horizontal.
13. Given $f(x) = x^3 - 6x^2 + 9x - 5$.



- (a) Find the slope of the tangent line to the curve at $x = -2$. (You may assume that $df/dx = 3x^2 - 12x + 9$.)
- (b) What is the equation of this tangent line?
- (c) What is the equation of the **normal line** at $x = -2$.
- (d) Find all points where the curve has a horizontal tangent.
14. Does the curve $y = x^3 + x + 1$ ever have a horizontal tangent line? If so, where?

15. Let $G(x) = 2^{bx}$, where b is a non-zero constant. Albertine, our friend, informs us that $d/dx (2^{bx}) = b 2^{bx} \ln 2$.

Find $G^{(2019)}(x)$. Show your reasoning! An answer with no justification will earn little or no credit.

16. Let $f(x) = x|x|$ be defined on the real line. Is f differentiable at $x = 0$? True or False? Explain.

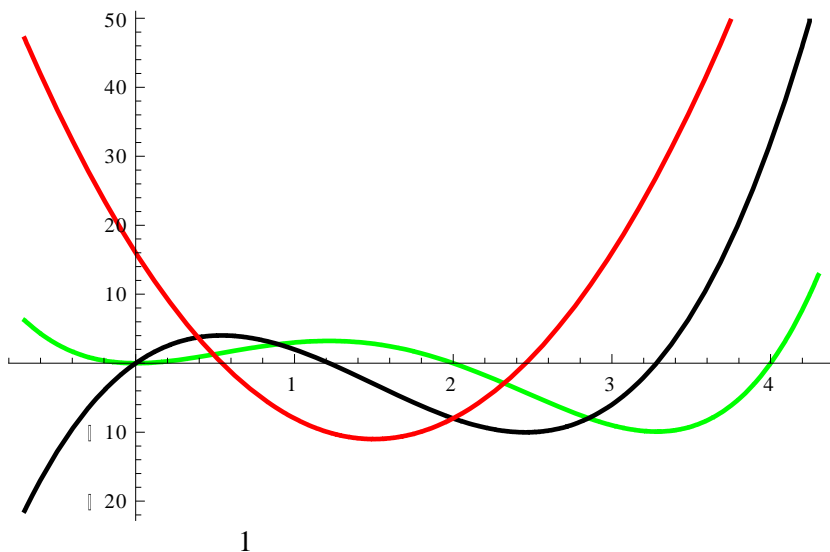
17. Assume that Charlotte, who chooses to live on the y -axis, is located at

$$y(t) = 3 \cos t + 4 \sin t \text{ cm at time } t \text{ (measured in minutes).}$$

You may assume that $\frac{d}{dx} \sin x = \cos x$ and that $\frac{d}{dx} \cos x = -\sin x$

- (a) Find her *position* at times $t = 0$, $t = \pi/2$, and $t = \pi$.
- (b) Find her *velocity* when $t = 0$, $t = \pi/2$, and $t = \pi$ minutes.
- (c) Find her *acceleration* when $t = 0$, $t = \pi/2$, and $t = \pi$.

18. Archy lives on the x -axis. Graphs of his *position*, *velocity* and *acceleration* during the time interval $-0.7 < t < 4.3$ appear below. Which is which? Explain.



19. Let $f(x) = x^4 - ax^2$.

(a) Find all possible critical points of f in terms of a .

You may assume that $f'(x) = 4x^3 - 2ax$.

(b) If $a < 0$, how many critical points does f have?

(c) If $a > 0$, find the x and y coordinates of all critical points of f .

20. Given $f(x) = x^6 - 3x^5$ on the interval $[-1, 4]$.

You may assume that $f'(x) = 6x^5 - 15x^4$.

- (a) Find all critical points of f .
- (b) Determine on which intervals f is increasing.
- (c) Find and classify all local and global extrema of f .
- (d) Sketch the graph of f using the above information.

21. Given $g(x) = \frac{x}{x+1}$.

You may assume that $g'(x) = \frac{x}{(x+1)^2}$.

- (a) What is the domain of g ?
- (b) Find all critical points of g .
- (c) Determine on which intervals g is increasing.
- (d) Find and classify all local and global extrema of g .
- (e) Sketch the graph of f using the above information.

22. Elphaba the squirrel has been involved in some questionable activity of late and hence is being very cautious. She has made eye contact with a human standing near her multiple times and is getting anxious that the human is observing her. Let $f(x)$ be Elphaba's anxiety (in "anxious units") after making eye contact with the human for a total of x seconds. Elphaba will panic and run when her anxiety reaches 100 anxious units. From across the room, the human, Erin, is in fact observing Elphaba while pretending to read a newspaper. The total amount of time Elphaba has spent making eye contact with Erin is a function of the number of times that Erin looks up from the newspaper. Let $g(n)$ be the total amount of time, in seconds, that Erin and Elphaba have spent making eye contact if Erin has looked up from her newspaper n times.

a. [2 points] Using a complete sentence, give a practical interpretation of the expression $f^{-1}(3) = 10$. Be sure to include units.

b. [3 points] Below is the first part of a sentence that will give a practical interpretation of the equation

$$f'(25) = 2.$$

Complete the sentence so that the practical interpretation can be understood by someone who knows no calculus. Be sure to include units in your answer.

If Elphaba has already made eye contact with Erin for a total of 25 seconds and she makes eye contact for an additional 0.3 seconds, then

c. [2 points] Given that $(f^{-1})'(99) = 7$ and $f(62) = 99$, approximate the total length of time Elphaba has to spend making eye contact with Erin before she will panic and run.

d. [3 points] Which of the following sentences gives a correct interpretation of the quantity $g^{-1}(f^{-1}(50))$? Circle the ONE best answer.

i. When Erin has looked up from her newspaper 50 times, Elphaba's anxiety is at $g^{-1}(f^{-1}(50))$ anxious units.

ii. When Erin has looked up from her newspaper 50 times, Erin and Elphaba have spent $g^{-1}(f^{-1}(50))$ seconds making eye contact.

iii. If Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times then Elphaba's anxiety is 50 anxious units.

iv. If Erin and Elphaba have made eye contact for a total of 50 seconds then Erin has looked up from her newspaper $g^{-1}(f^{-1}(50))$ times.

v. When Erin and Elphaba have made eye contact for a total of 50 seconds then

23. Your local cable internet provider has discovered that the strength (measured in Watts) of the signal generated at their broadcast station decreases fairly rapidly as it travels over their wires. They are concerned about this because subscribers must receive at least a 12-Watt signal in order for their systems to work. Engineers have calculated that $S = f(d) = 160(0.64)^d$, where S is the signal strength, and d represents distance (in miles) from the broadcast station.

(a) Translate the statement " $f^{-1}(x) = 11$ " into plain English. Is this statement true or false?

(b) What percentage of the signal is lost over each mile of cable?

(c) What does the "160" tell you, in real world terms? (Answer in a complete sentence.)

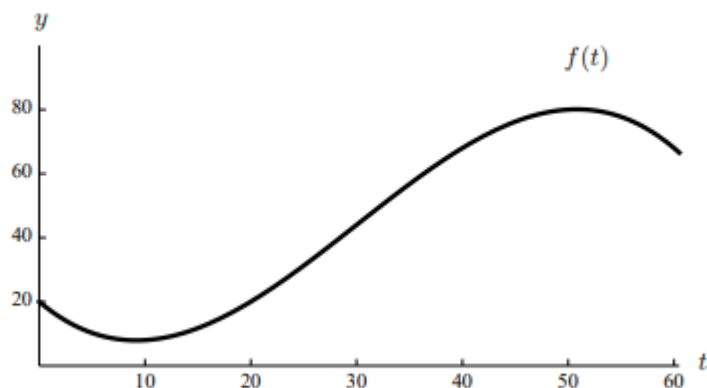
(d) How far from the station can one live and still receive the service?

24. Your pet bird is flying in a straight path toward you and away from you for a minute.

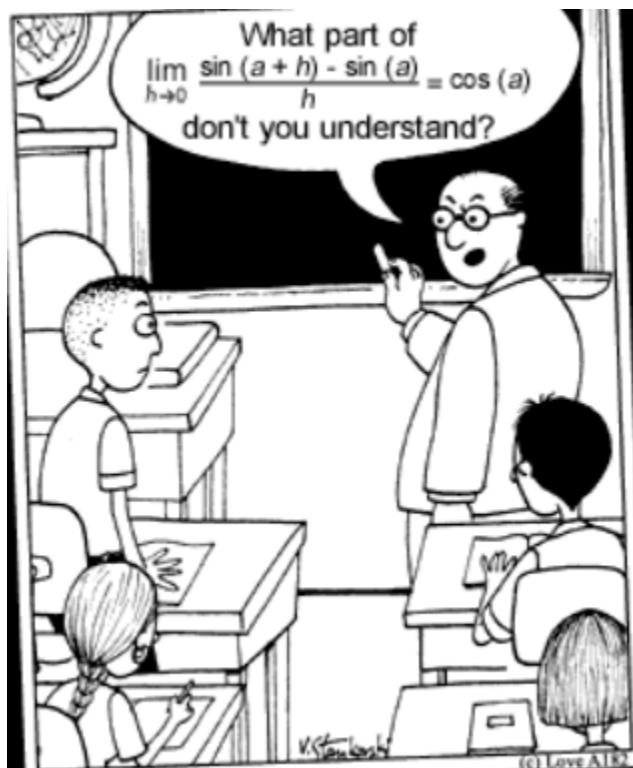
After t seconds, she is $f(t)$ feet away from you, where

$$f(t) = \frac{-t(t-20)(t-70)}{500} + 20, \quad 0 \leq t \leq 60.$$

A graph of $y = f(t)$ is shown here.



- [3 points] Without doing any calculations, determine which is greater: the average velocity of the bird over the entire minute, or her instantaneous velocity after 30 seconds. Explain, referring to the graph.
- [3 points] Calculate the exact value of the average velocity of the bird over the entire minute.
- [4 points] Write an explicit expression for the velocity of the bird at time t using the limit definition of velocity. Final answers containing the letter f will receive no credit. Do not evaluate your expression.
- After a minute, you frighten the bird, and she flies away at 9 feet/sec. Write a formula for a continuous function $f(t)$ describing the distance between you and the bird for $0 \leq t \leq 180$.



When I was four years old they tried to test my IQ. They showed me a picture of three oranges and a pear. They asked me, "Which one is different and does not belong?" They taught me different was wrong.

- Ani DiFranco

I turn away with fright and horror from the lamentable evil of functions that do not have derivatives.

- Charles Hermite (in a letter to Thomas Jan Stieltjes)