# DISCUSSION QUESTIONS: 20 SEPT 2019

## SHORTCUTS


"Don't worry... I know a short cut."

## REVIEW

**1.** (U. Michigan) Use the limit definition of the derivative to write an explicit expression for r′(3) where r(t) = $(t + 5)^{2t}$ . Do not simplify or evaluate the limit. Your answer should not include the letter r.

**2.** (U. Michigan) Two housecats, Jasper and Zander, escape from their house at the same time and travel along a straight line between their house and a tree. Let J(t) (respectively Z(t)) be Jasper's (respectively Zander's) distance, in feet, from the tree *t* seconds after escaping.

The table below shows some of the values of J(t) and Z(t). Assume that J(t) is invertible.

| $t$ | 6 | 17 | 22 | 31 | 37 |
|---|---|---|---|---|---|
| $J(t)$ | 41 | 33 | 21 | 14 | 2 |
| $Z(t)$ | 39 | 32 | 31 | 36 | 43 |

a. What is Jasper's average velocity for $6 \le t \le 22$? Be sure to include units.

b. Estimate Z ′(31). Remember to show your work.

c. Circle the one statement below that is best supported by the equation $Z(J^{-1}(8) - 4) = 34$.

i. 34 seconds after escaping, Zander is 4 feet closer to the tree than Jasper was 8 seconds after escaping.

ii. Four seconds before Jasper is 8 feet from the tree, Zander is 34 feet from the tree.

iii. When Jasper is 4 feet further from the tree than he was 8 seconds after escaping, Zander is 34 feet from the tree.
iv. When Jasper is 4 feet closer to the tree than he was 8 seconds after escaping, Zander is 34 feet from the tree. v. Four seconds after Jasper is 8 feet from the tree, Zander is 34 feet from the tree.

d. Circle the one statement below that is best supported by the equation $(J^{-1})'(3) = -0.2$.

i. In the third second after leaving the house, Jasper travels about 0.2 feet.
ii. When Jasper is 3 feet from the tree, he is traveling about 0.2 feet/second slower than he was one foot earlier.
iii. Jasper gets about 1.5 feet closer to the tree during the third second after leaving the house.
iv. It takes Jasper about one-tenth of a second to go from 3 feet to 2.5 feet from the tree.
v. One-half of a second before Jasper was 3 feet from the tree, he was about 2.9 feet from the tree.

# SHORTCUTS

**1)** State and prove the rules of differentiation, including the power, product and quotient rules.

**2)** Using the short cuts of differentiation *when appropriate*, compute the derivative of each of the following functions.

$(A)$ $y = 2019 + 5x - \pi x^4 + e^4$

$(B)$ $y = x \sin x$

$(C)$ $y = \dfrac{x+3}{x+7}$

$(D)$ $y = \dfrac{x}{\sin x}$

$(E)$ $y = \dfrac{\cos x}{x^3+9}$

$(F)$ $y = (x^2+4x-1)(x^3+5x^4-x^3+x^2+3x+13)$

$(G)$ $y = \sin^2 x$

$(H)$ $y = (x^2+5x+1)^2$

**3)** (a) Find the equations of the *tangent* and *normal lines* to the curve

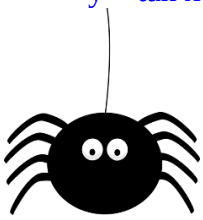
$$y = \frac{x-4}{x+1} \quad at\ x = 3.$$

(b) Find the equations of the *tangent* and *normal* lines to the curve

$y = \sin x$ at $x = \pi/4$.

**4)** Using appropriate shortcuts, find formulas for the derivatives of

$y = \tan x$ and $y = \sec x$.



**5)** Charlotte, the spider, dances along the x-axis according to the rule $x(t) = t^3 - 3t + 5$. (Here time is measured in *seconds* and distance in *cm*.)

(a) Find Charlotte's *velocity* at time t = 2 sec.

(b) Find Charlotte's *acceleration* at time t = 2 sec.

**6)** Sketch the curve $y = x^2(x - 2)^2$. Over which interval(s) is the graph *rising? falling?* Locate any local maxima or minima.

**7)** Sketch the curve $y = \frac{x-4}{x+1}$ (cf. problem II a). Over which interval(s) is the graph *rising*? *falling*? Locate any *local maxima* or *minima*.

**8)** Sketch the curve $y = xe^x$. Over which interval(s) is the graph rising? *falling*? Locate any local maxima or minima.

**9)** Sketch the curve $y = \frac{x-3}{x^2+1}$. Over which interval(s) is the graph rising? *falling*? Locate any local maxima or minima.

**10)** Consider the curve $y = b + c \sin x$. For each of the following values of *b* and *c*, determine when the graph is rising and when it is falling:

(a) b = 3, c = 1
(b) b = c = 1
(c) b = 1, c = 2

**11)** Sketch the curve $y = \frac{1}{x} + x^2$ over the interval (0, ∞). Over which interval(s) is the graph *rising*? *falling*? Locate any local maxima or minima.

*What Romantic terminology called genius or talent or inspiration is nothing other than finding the right road empirically, following one's nose, taking shortcuts.*

- Italo Calvino (1923 – 1985)