DISCUSSION QUESTIONS: 23 SEPT 2019

SHORTCUTS CONTINUED

REVIEW

1. *(U. Michigan)* Use the limit definition of the derivative to write an explicit expression for $g'(5)$ where $g(x) = \left(1 + \frac{2}{x}\right)^{\ln(5x+4)}$. Do not simplify or evaluate the limit. Your answer should not include the letter *g*.
2. *(U. Michigan)* Albertine is living on the 37th floor of a fancy building in Chicago. She wants to get rid of an ancient (very energy inefficient) refrigerator that was in the building before alterations were made to the apartment.
   The box will not fit through the new doors of the apartment, so the refrigerator must be pushed down a rather rickety ramp out the window. The ramp is 350 feet long. Below is a table showing the distance from the window along the ramp at given times:

time (seconds)	0	2	4	6	8	10	12	14	16
distance from window (feet)	0	3.9	18.6	43.1	79.4	122.5	174.6	240.1	313.6

Suppose s(t) = d is the distance from the window, in feet, as a function of time, *t*, in seconds.

(a) Compute the *average velocity* of the refrigerator over the time interval $4 \le t \le 12$.
(b) *Approximate the instantaneous velocity* of the refrigerator when t = 8 seconds.

SHORTCUTS (REVISED)

1) State and prove the rules of differentiation, including the power, product, and quotient rules.

2) Prove that $\frac{d}{dx}\sin x = \cos x$.

3) Using the short cuts of differentiation *when appropriate*, compute the derivative of each of the following functions.

(A) $y = 2019 + 5x - \pi x^4 + e^4$

(B) $y = x\sin x$

(C) $y = \dfrac{x+3}{x+7}$

(D) $y = \dfrac{x}{\sin x}$

(E) $y = \dfrac{\cos x}{x^3 + 9}$

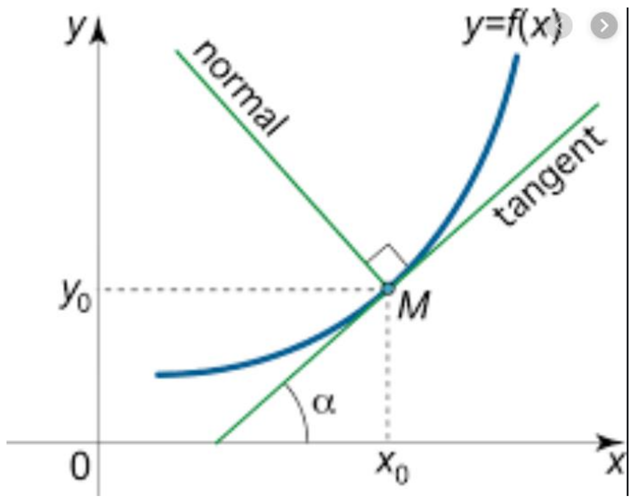
$(F)\quad y=(x^2+4x-1)(x^3+5x^4-x^3+x^2+3x+13)$

$(G)\quad y=\sin^2 x$

$(H)\quad y=(x^2+5x+1)^2$

4) (a) Find the equations of the *tangent* and *normal lines* to the curve

$$y=\frac{x-4}{x+1}\quad at\ x=3.$$



(b) Find the equations of the *tangent* and *normal* lines to the curve

$y = \sin x$ at $x = \pi/4$.

5) Using appropriate shortcuts, find formulas for the derivatives of

$y = \tan x$ and $y = \sec x$.

6) Charlotte, the spider, dances along the x-axis according to the rule $x(t) = t^3 - 3t + 5$. (Here time is measured in *seconds* and distance in *cm*.)

(a) Find Charlotte's *velocity* at time t = 2 sec.

(b) Find Charlotte's *acceleration* at time t = 2 sec.

7) Sketch the curve $y = x^2(x-2)^2$. Over which interval(s) is the graph *rising? falling?* Locate any local maxima or minima.

8) Sketch the curve $y = \frac{x-4}{x+1}$ (cf. problem II a). Over which interval(s) is the graph *rising*? *falling*? Locate any *local maxima* or *minima*.

9) Give a plausible argument that $\frac{d}{dx} e^x = e^x$.

10) Sketch the curve $y = xe^x$. Over which interval(s) is the graph rising? *falling*? Locate any local maxima or minima.

11) Sketch the curve $y = \frac{x-3}{x^2+1}$. Over which interval(s) is the graph rising? *falling*? Locate any local maxima or minima.

12) Consider the curve $y = b + c \sin x$. For each of the following values of *b* and *c*, determine when the graph is rising and when it is falling:

(a) b = 3, c = 1
(b) b = c = 1
(c) b = 1, c = 2

13) Sketch the curve $y = \frac{1}{x} + x^2$ over the interval $(0, \infty)$. Over which interval(s) is the graph *rising*? *falling*? Locate any local maxima or minima.

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