DISCUSSION QUESTIONS: 25 SEPTEMBER 2019

SHORTCUTS APPLIED TO CURVE SKETCHING

1) (a) Find the equations of the *tangent* and *normal lines* to the curve

$$y = \frac{x-4}{x+1} \quad at\ x = 3$$

(b) Sketch the curve $y = \frac{x-4}{x+1}$ $at\ x = 3$

Over which interval(s) is the graph *rising*? *falling*? Locate any *local maxima* or *minima*.

2) Find the equations of the *tangent* and *normal* lines to the curve

$y = \sin x$ at $x = \pi/4$.

3) Using appropriate shortcuts, find formulas for the derivatives of

$y = \tan x$ and $y = \sec x$.

4) Sketch the curve $y = x\,(x - 1)^2$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

5) Sketch the curve $y = \frac{x^3}{1+x^2}$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

6) Sketch the curve $y = x^3(x - 2)^2$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

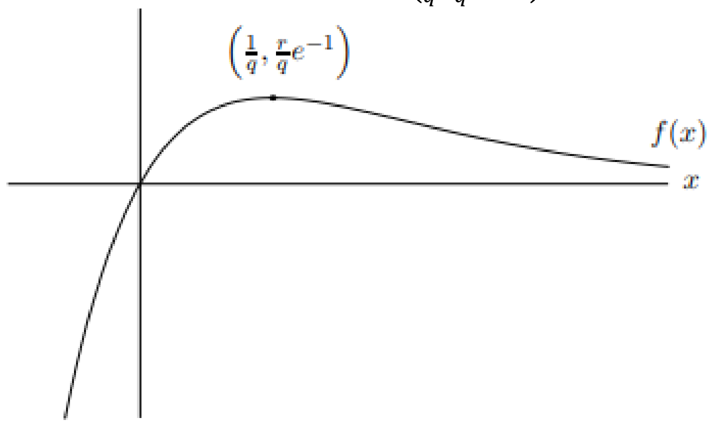
7) Sketch the curve $y = x\,e^x$. Over which interval(s) is the graph rising? *falling?* Locate any local maxima or minima.

8) Sketch the curve $y = \frac{1}{x} + x^2$ over the interval $(0, \infty)$. Over which interval(s) is the graph *rising*? *falling*? Locate any local maxima or minima.

9) *[U. of Michigan]* Below is the graph of the function

$$f(x) = r\,x\,e^{-qx}$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$ passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}\,e^{-1}\right)$, as shown in the graph below:



a. Justify, using the first-derivative test that the point P is a local maximum.

b. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain [0, 1]? (If $f(x)$ does not have a global maximum on this domain, say "no")

c. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum," and similarly if $f(x)$ does not have a global minimum

d. Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum, and each minimum is a minimum.