DISCUSSION QUESTIONS: 30 SEPTEMBER 2019

SHORTCUTS APPLIED TO CURVE SKETCHING

1) What is the "first-derivative test"?

2) *[Stewart]* Find all critical points and classify each:

(a) $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + 1$

(b) $F(x) = x^4 - 2x^2 + 1$

(c) $g(x) = x^4 (x - 1)$

(d) $L(x) = x^7 - x^5 + 3$

(e) $y = \frac{x}{x^2+1}$

3) (a) Find equations of the *tangent* and *normal lines* to the curve

$$y = \frac{x-4}{x+1} \quad at\ x = 3$$

(b) Sketch the curve $y = \frac{x-4}{x+1}$. Over which interval(s) is the graph *rising*? *falling*? Locate any *local maxima* or *minima*.

4) Find the equations of the *tangent* and *normal* lines to the curve

$y = \sin x$ at $x = \pi/4$.

5) Sketch the curve $y = x(x-1)^2$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

6) Sketch the curve $y = \frac{x^3}{1+x^2}$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

7) Sketch the curve $y = x^4(x-2)^2$. Over which interval(s) is the graph *rising? falling?*

Locate any local maxima or minima.

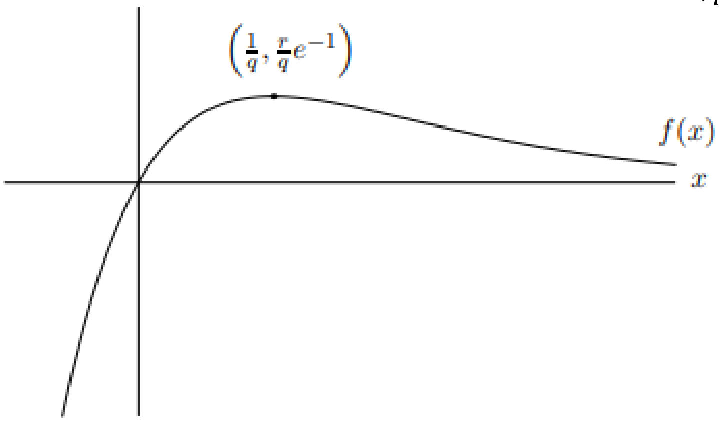
8) Sketch the curve $y = x\,e^x$. Over which interval(s) is the graph rising? *falling?* Locate any local maxima or minima.

9) Sketch the curve $y = \frac{1}{x} + x^2$ over the interval $(0, \infty)$. Over which interval(s) is the graph *rising*? *falling*? Locate any local maxima or minima.

10) *[U. of Michigan]* Below is the graph of the function

$$f(x) = r\,x\,e^{-qx}$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$ passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}e^{-1}\right)$, as shown in the graph below:

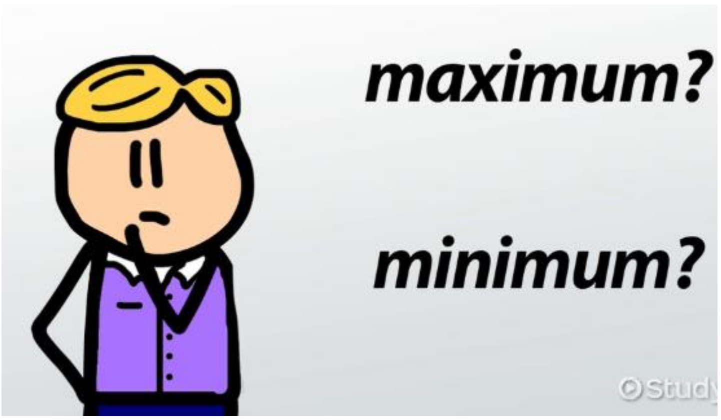


a. Justify, using the first-derivative test that the point P is a local maximum.

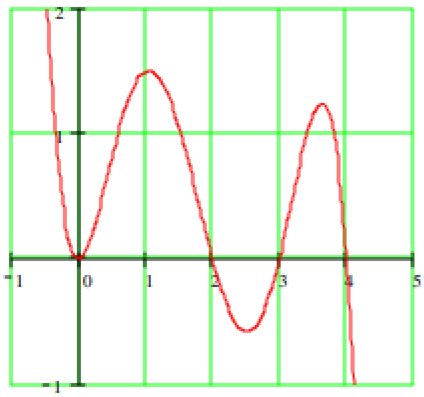
b. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain [0, 1]? (If $f(x)$ does not have a global maximum on this domain, say "no")

c. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum," and similarly if $f(x)$ does not have a global minimum

d. Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum, and each minimum is a minimum.



11) *[U. of Michigan]* The graph in the figure below is the graph of f ′ (x) (i.e., the graph of the derivative of f). [Note: all questions refer to f, not f ′.]



Graph of the derivative of f

(a) Determine all values of x for which:

(i) f has critical point(s) ____________________

(ii) f has local maximum(s) ____________________

(iii) f has local minimum(s) ____________________

(b) Give the largest interval over which f is increasing

12) *[Whitman College]* For each of the following functions, find and classify all critical points.

a) $y = x^2 - x$

b) $y = 2 + 3x - x^3$

c) $y = x^3 - 9x^2 + 24x$

d) $y = x^4 - 2x^2 + 3$

e) $y = 3x^4 - 4x^3$

f) $y = \frac{x^2-1}{x}$

g) $y = 3x^2 - \frac{1}{x^2}$

h) $y = \cos(2x) - x$

i) $y = \frac{x^3}{x+1}$

Swift Introduction to the Game of Anti-derivatives

For each of the following functions determine an anti-derivative:

$\sin x$, $2x$, $3x^2$, $\cos x$, x^8, $\sec^2 x$, $7x^2 + 9x^3 + e$, $(\sec x \tan x)$, $1 + e^x$