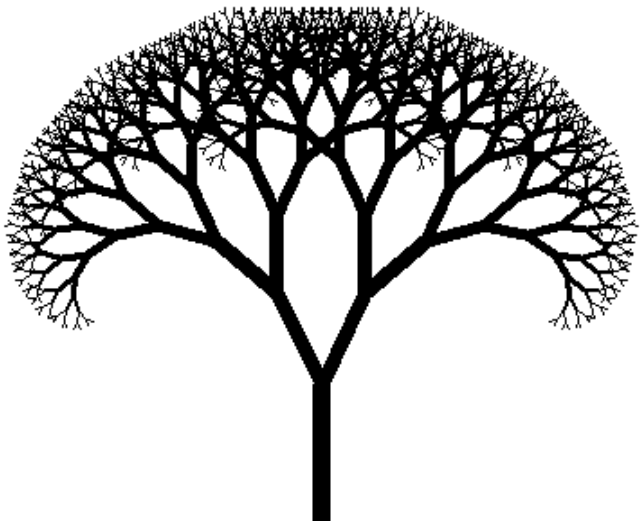
DISCUSSION: 4 SEPTEMBER 2019

limits & continuity



Fractal tree

I Evaluate each of the following limits or explain why the limit fails to exist.

1. $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-5x+6}$ 2. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}$ 3. $\lim\limits_{x\to 1}\dfrac{x^4-1}{x^2-1}$ 4. $\lim\limits_{x\to 1}\dfrac{x+9}{x^2-4}$

5. $\lim\limits_{x\to 1}\dfrac{x^4-1}{x^3-1}$ 6. $\lim\limits_{x\to 0}\dfrac{|x|}{x}$ 7. $\lim\limits_{x\to 16}\dfrac{\sqrt{x}-4}{x-16}$ 8. $\lim\limits_{x\to 1}\dfrac{\frac{1}{x}-1}{x-1}$

9. $\lim\limits_{x\to 4}\sqrt{\dfrac{x+5}{x+12}}$ 10. $\lim\limits_{x\to 1}\dfrac{x^2-1}{(x-1)^3}$

II State the *limit laws*, viz.

$\lim cf(x) =$

$\lim\big(f(x)+g(x)\big) =$

$\lim f(x)g(x) =$

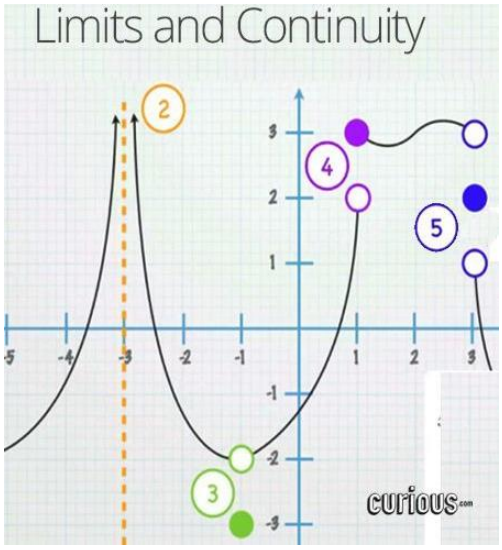
$\lim\dfrac{f(x)}{g(x)} =$

State the necessary conditions for which each rule is true.

III Define *continuity* of a function y = f(x) at x = a. What does it mean for a function to be *continuous*?

IV (a) For each of the four types of discontinuity (*removable, infinite, jump, essential*) give several examples.

(b) For the graph below, characterize each of the four discontinuities.



(c) Give an example of an *essential discontinuity.*

V Consider each of the following functions at the given point on the x-axis. Does the function have a *continuous extension* at the given point? Explain.

1. $f(x)=\dfrac{x-2}{x-3},\ x=3$

2. $G(x)=\dfrac{x^2-9}{x-3},\ x=3$

3. $H(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=4$

4. $g(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=1/3$

VI For which value of *a* is the following function *continuous everywhere?*

$$f(x)=\begin{cases} x^2-1 & for\ x<3 \\ 2ax & for\ x\geq 3 \end{cases}$$

VII For which values of *a* and *b* is the following function *continuous everywhere?*

$$g(x)=\begin{cases} ax+2b & for\ \ x\le 0 \\ x^2+3a-b & for\ \ 0<x\le 2 \\ 3x-5 & for\ \ x>2 \end{cases}$$

Exercises from Stewart

1. Given that

$$\lim_{x\to 2} f(x)=4 \qquad \lim_{x\to 2} g(x)=-2 \qquad \lim_{x\to 2} h(x)=0$$

find the limits that exist. If the limit does not exist, explain why.

a. $\lim_{x\to 2}\left[f(x)+5g(x)\right]$

b. $\lim_{x\to 2}\left[g(x)\right]^3$

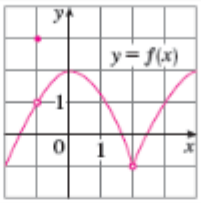
c. $\lim_{x\to 2}\sqrt{f(x)}$

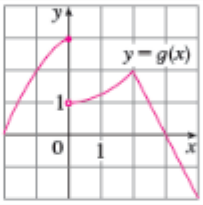
d. $\lim_{x\to 2}\dfrac{3f(x)}{g(x)}$

e. $\lim_{x\to 2}\dfrac{g(x)}{h(x)}$

f. $\lim_{x\to 2}\dfrac{g(x)\,h(x)}{f(x)}$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x \to 2} [f(x) + g(x)]$

b. $\lim_{x \to 0} [f(x) - g(x)]$

c. $\lim_{x \to -1} [f(x)\, g(x)]$

d. $\lim_{x \to 3} \dfrac{f(x)}{g(x)}$

e. $\lim_{x \to 2} [x^2 f(x)]$

f. $f(-1) + \lim_{x \to -1} g(x)$

3, 4, 5, 6, 7, 8 and 9 Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

3. $\lim_{x \to 3} (5x^3 - 3x^2 + x - 6)$

Answer

4. $\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$

5. $\lim_{t \to -2} \dfrac{t^4 - 2}{2t^2 - 3t + 2}$

Answer

6. $\lim_{u \to -2} \sqrt{u^4 + 3u + 6}$

7. $\lim_{x \to 8} (1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$

Answer

8. $\lim_{t \to 2} \left(\dfrac{t^2 - 2}{t^3 - 3t + 5} \right)^2$

9. $\lim_{x \to 2} \sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 and 32 Evaluate the limit, if it exists.

11. $\lim_{x \to 5} \dfrac{x^2 - 6x + 5}{x - 5}$

12. $\lim_{x \to -3} \dfrac{x^2 + 3x}{x^2 - x - 12}$

13. $\lim_{x \to 5} \dfrac{x^2 - 5x + 6}{x - 5}$

14. $\lim_{x \to 4} \dfrac{x^2 + 3x}{x^2 - x - 12}$

15. $\lim_{t \to -3} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$

16. $\lim_{x \to -1} \dfrac{2x^2 + 3x + 1}{x^2 - 2x - 3}$

17. $\lim_{h \to 0} \dfrac{(-5 + h)^2 - 25}{h}$

18. $\lim_{h \to 0} \dfrac{(2 + h)^3 - 8}{h}$

19. $\lim_{x \to -2} \dfrac{x + 2}{x^3 + 8}$

20. $\lim_{t \to 1} \dfrac{t^4 - 1}{t^3 - 1}$

21. $\lim_{h \to 0} \dfrac{\sqrt{9 + h} - 3}{h}$

22. $\lim_{u \to 2} \dfrac{\sqrt{4u + 1} - 3}{u - 2}$

23. $\lim_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

24. $\lim_{h \to 0} \dfrac{(3 + h)^{-1} - 3^{-1}}{h}$

25. $\lim_{t \to 0} \dfrac{\sqrt{1 + t} - \sqrt{1 - t}}{t}$

26. $\lim_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

27. $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

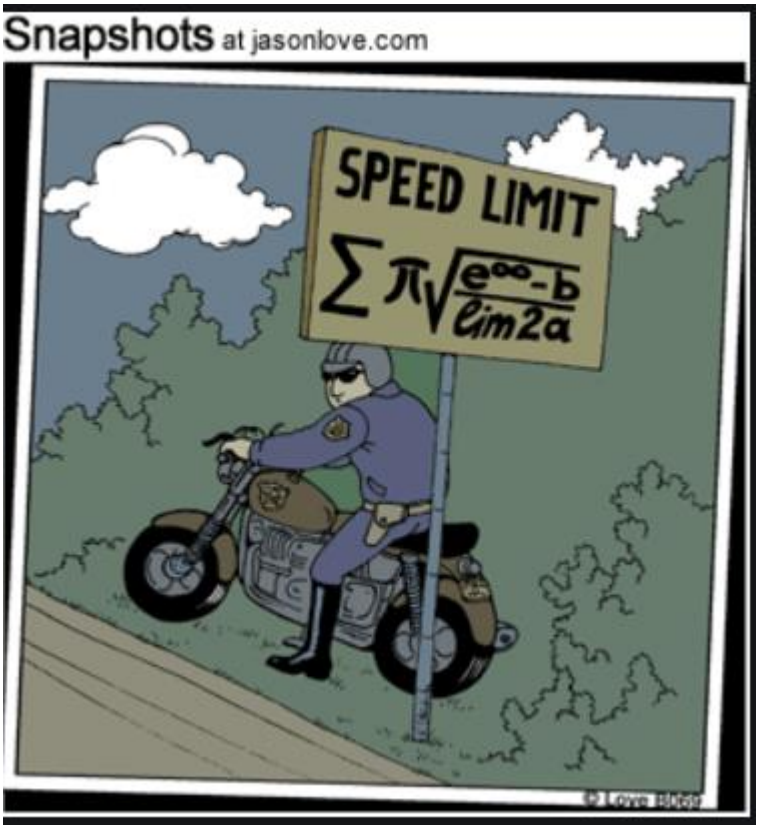
28. $\lim_{x \to 2} \dfrac{x^2 - 4x + 4}{x^4 - 3x^2 - 4}$

29. $\lim_{t \to 0} \left(\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$

30. $\lim_{x \to -4} \dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

31. $\lim_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

32. $\lim_{h \to 0} \dfrac{\dfrac{1}{(x + h)^2} - \dfrac{1}{x^2}}{h}$



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