DISCUSSION: 6 SEPTEMBER 2019

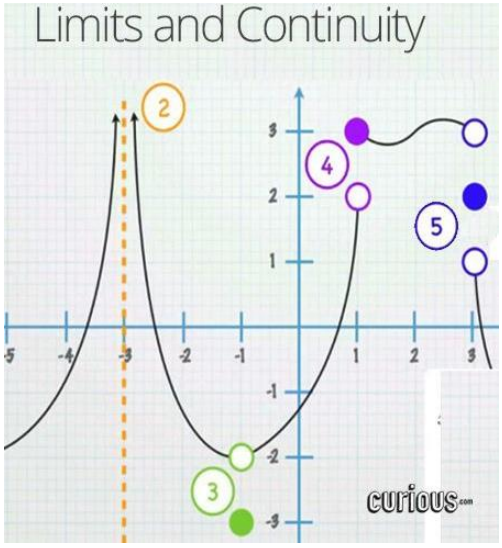
Continuity:

Intermediate Value Theorem; Squeeze Theorem

1. Define *continuity* of a function y = f(x) at x = a. What does it mean for a function to be *continuous*?

2. (a) For each of the four types of discontinuity (*removable, infinite, jump, essential*) give several examples.

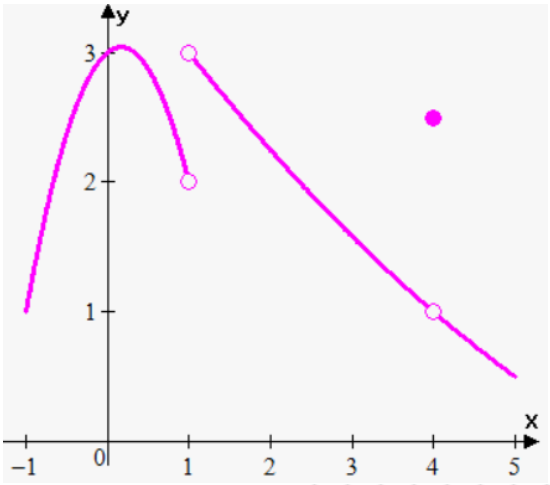
(b) For the graph below, characterize each of the four discontinuities.

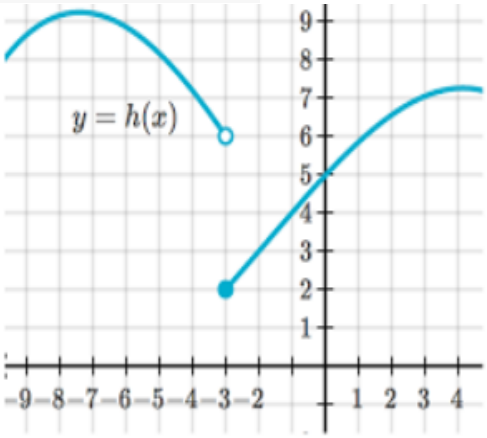


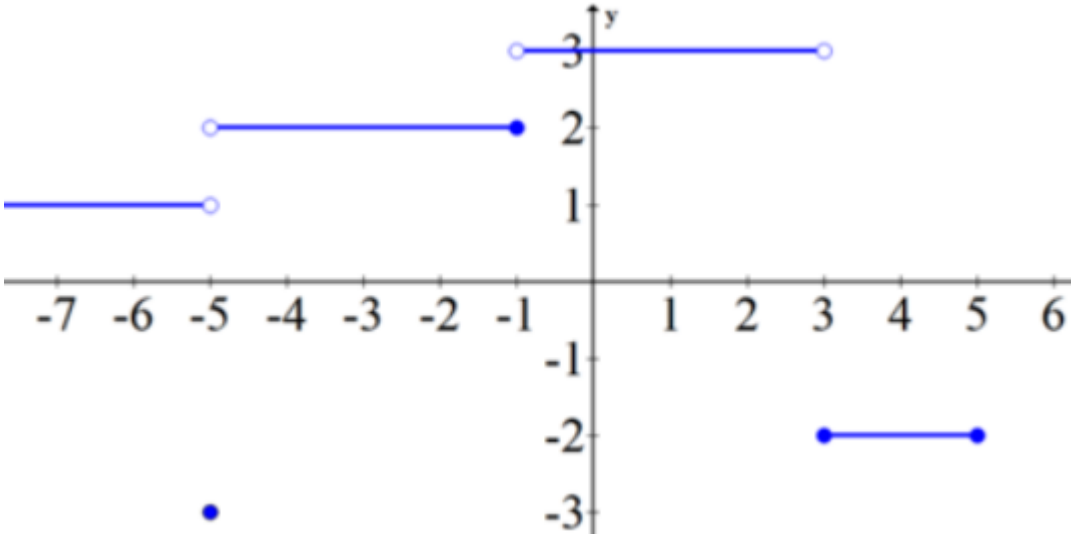
(c) Give an example of an *essential discontinuity.*

3. What is meant by "*one-sided*" limit?

For each of the following graphs, identify and compute one-sided limits at points of discontinuity



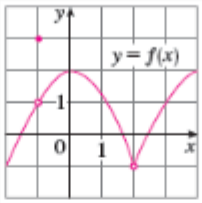


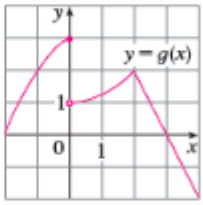


What is the relationship between one-sided limits and limit? What does this mean in terms of continuity?

4. (Stewart)

The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x\to 2}[f(x)+g(x)]$

b. $\lim_{x\to 0}[f(x)-g(x)]$

c. $\lim_{x\to -1}[f(x)\,g(x)]$

d. $\lim_{x\to 3}\dfrac{f(x)}{g(x)}$

e. $\lim_{x\to 2}[x^2 f(x)]$

f. $f(-1)+\lim_{x\to -1} g(x)$

5. Consider each of the following functions at the given point on the x-axis. Does the function have a *continuous extension* at the given point? Explain.

1. $f(x)=\dfrac{x-2}{x-3},\ x=3$

2. $G(x)=\dfrac{x^2-9}{x-3},\ x=3$

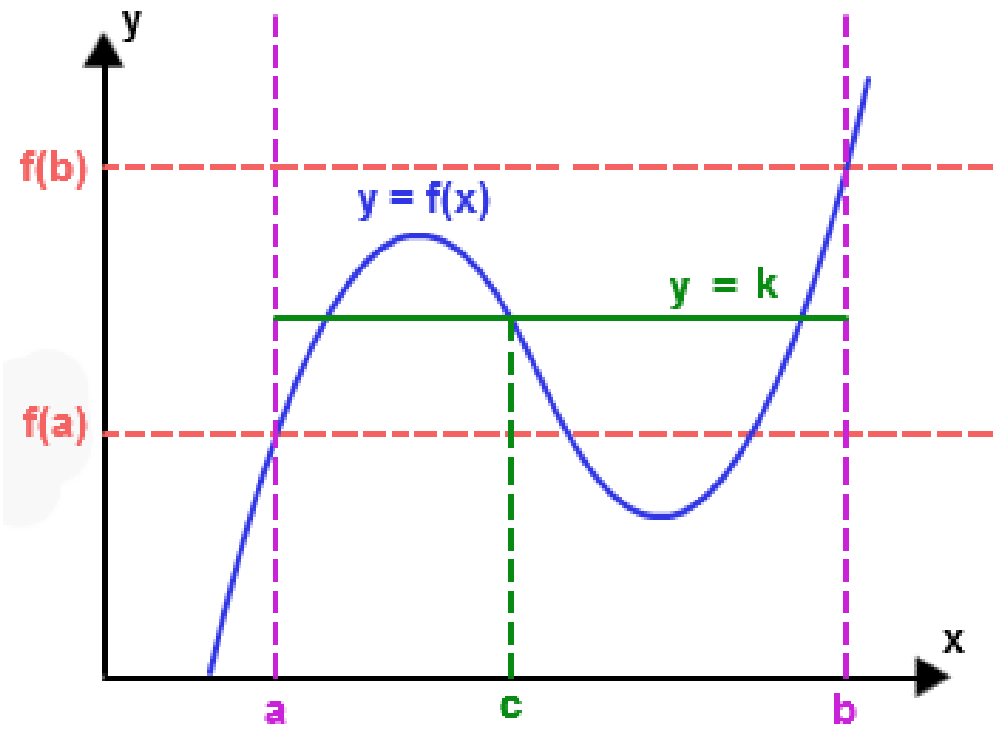
3. $H(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=4$

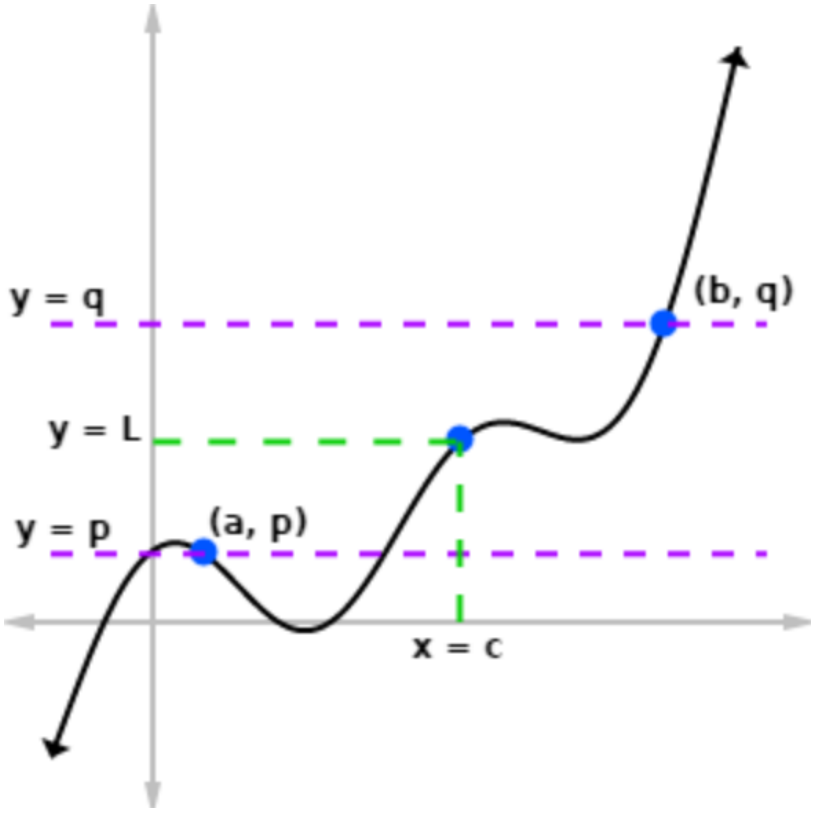
4. $g(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=1/3$

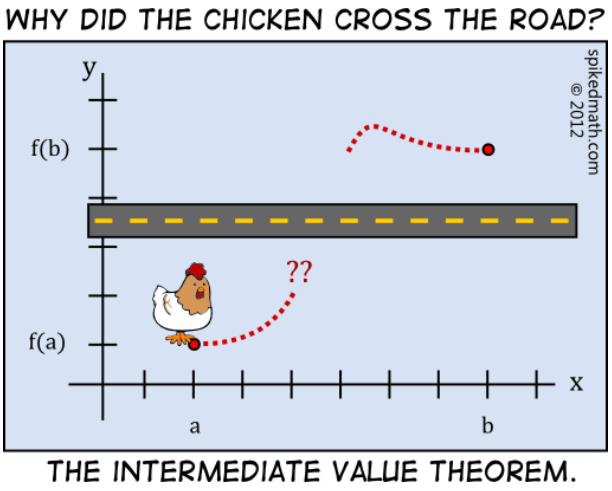
6. For which value of *a* is the following function *continuous everywhere?*

$$f(x) = \begin{cases} x^2 - 1 & for\ x < 3 \\ 2ax & for\ x \geq 3 \end{cases}$$

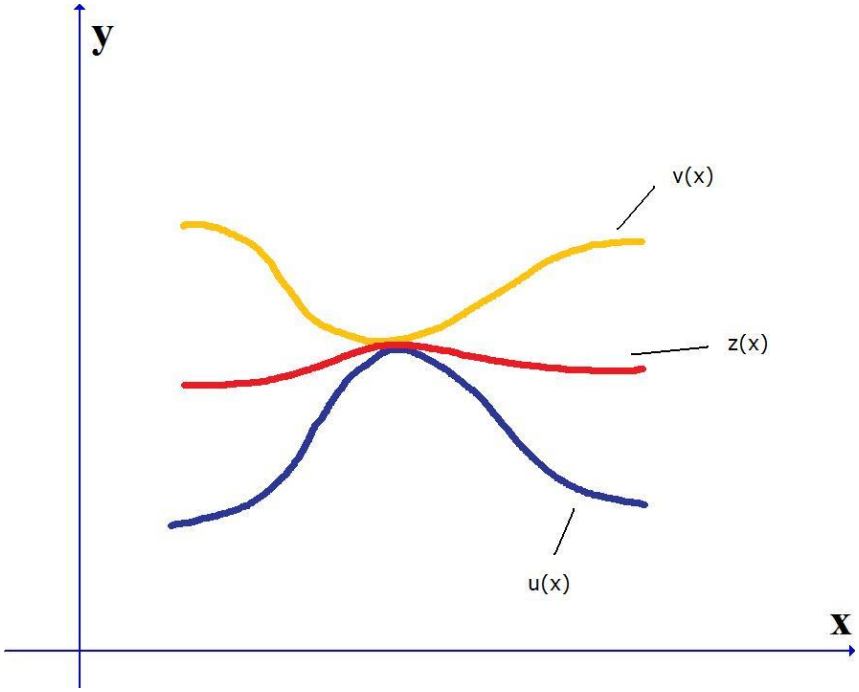
7. State the *Intermediate Value Theorem.* Using the IVT, prove that the polynomial $f(x) = x^4 + 4x^3 - 20x + 11$ *must* have a root between x = 0 and x =

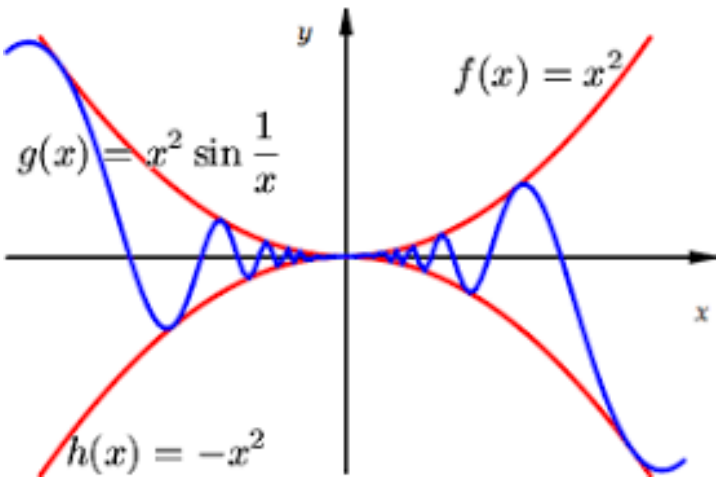

y
f(b)
y = f(x)
y = k
f(a)
x
a
c
b


y = q
(b, q)
y = L
y = p
(a, p)
x = c



8. State the *Squeeze Theorem (a.k.a. Sandwich Theorem, Pinching Theorem, Two Gendarmes Theorem, Two Policemen and a Drunk Theorem).*

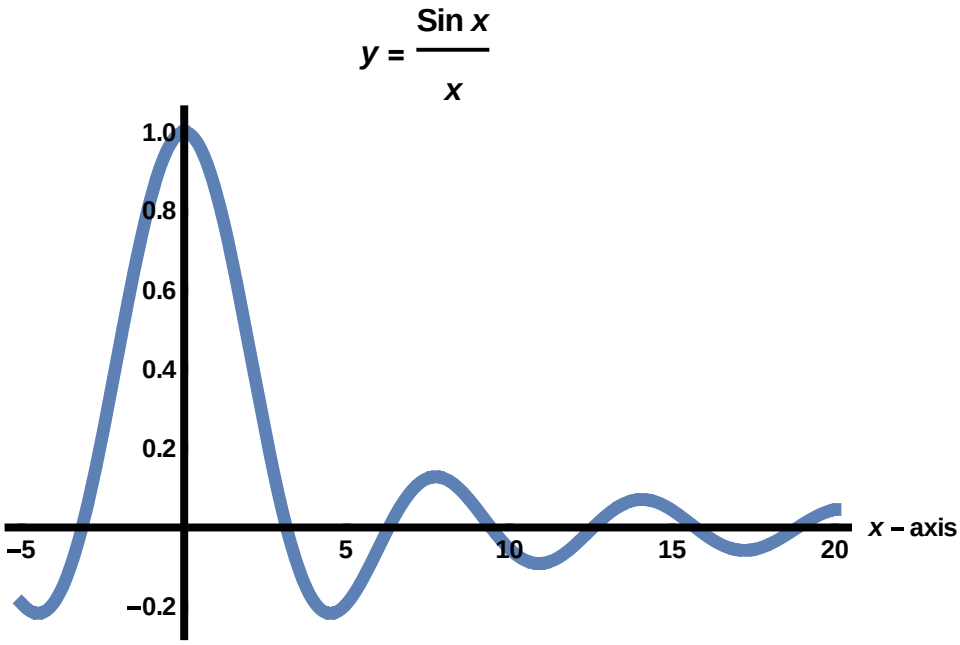




9. (a) Is the function $f(x) = \frac{\sin x}{x}$ *even* or *odd* or neither?

(b) Using the Sandwich Theorem prove that $\frac{\sin x}{x} \to 1$ *as* $x \to 0$.

(This result is the key to our being able to differentiate the trig functions.)



10. Using the Squeeze Theorem compute each of the following limits:

$(a)\ \lim_{x\to 0} x^8 \sin^4(1/x)$

$(b)\ \lim_{x\to 0} x^4 \cos(1/x)$

$(c)\ \lim_{x\to \infty} x\sin(1/x)$

$(d)\ \lim_{x\to\infty} \dfrac{x^2\cos(2x)+\sin^3(x^{2017})}{x^3+x+5}$

11. Infinite limits: Evaluate each of the following limits or explain why the limit fails to exist.

(a) $\lim_{x\to\infty} \frac{\sin x}{x}$

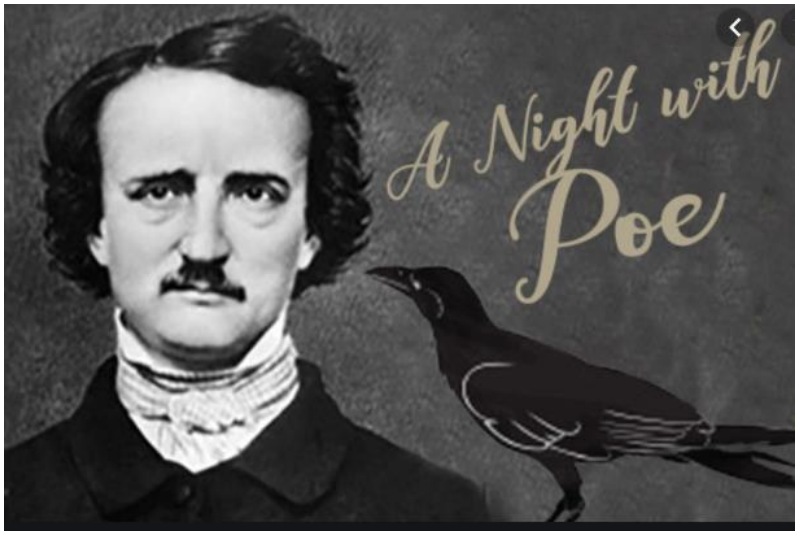
(b) $\lim_{x\to\infty} \frac{x^4+5x^2+2019}{(2x^2+13)^2}$

(c) $\lim_{x\to 1} \frac{x-3}{x^2+2x-4}$

(d) $\lim_{x\to 1} \frac{x-3}{x^2+2x-4}$

(e) $\lim_{h\to 4} \frac{x-4}{|4-x|}$

(f) $\lim_{x\to\infty} \frac{\sqrt{9x^2-3}}{7x^2+2x-4}$



"I could have clasped the red walls to my bosom as a garment of eternal peace. "Death," I said, "any death but that of the pit!" Fool! might I have not known that into the pit it was the object of the burning iron to urge me?"

— Edgar Allan Poe, **The Pit and the Pendulum**