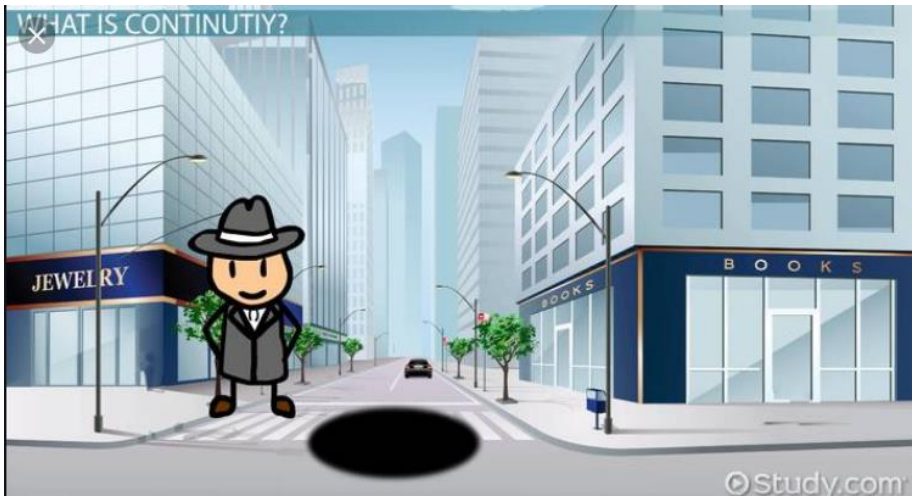
DISCUSSION: 9 SEPTEMBER 2019

Continuity:

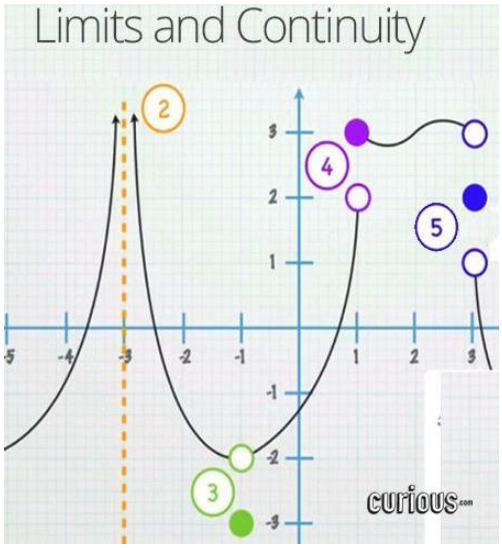
Intermediate Value Theorem; Squeeze Theorem

Trigonometric limits



1. (a) For each of the four types of discontinuity (*removable, infinite, jump, essential*) give several examples.

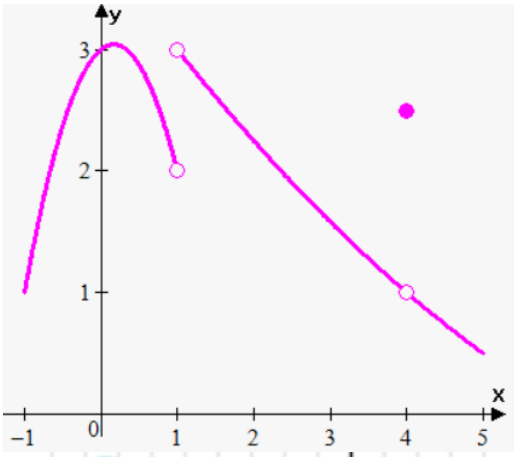
(b) For the graph below, characterize each of the four discontinuities.

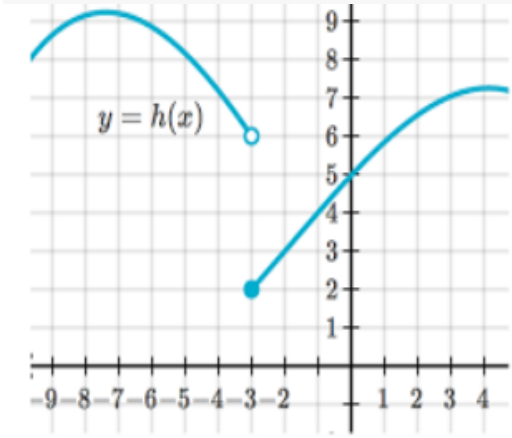


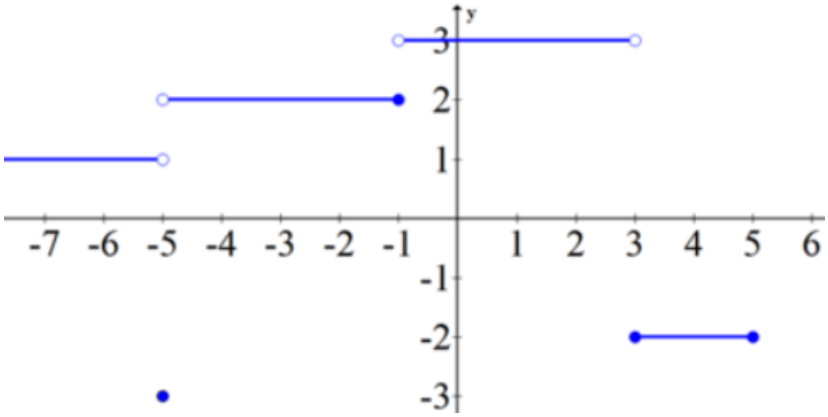
(c) Give an example of an *essential discontinuity.*

2. What is meant by "*one-sided*" limit?

For each of the following graphs, identify and compute one-sided limits at points of discontinuity







What is the relationship between one-sided limits and limit? What does this mean in terms of continuity?

3. Consider each of the following functions at the given point on the x-axis. Does the function have a *continuous extension* at the given point? Explain.

1. $f(x) = \dfrac{x-2}{x-3}, \quad x = 3$

2. $G(x) = \dfrac{x^2-9}{x-3}, \quad x = 3$

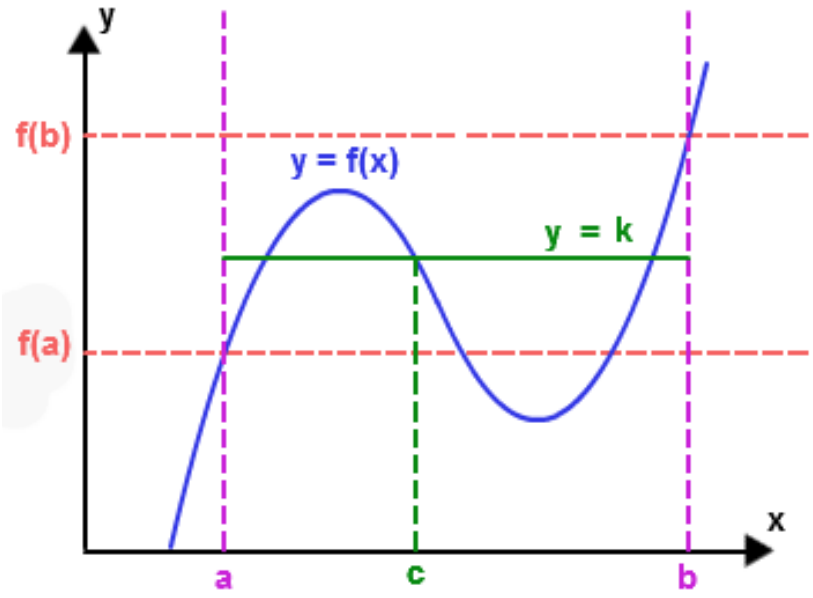
3. $H(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=4$

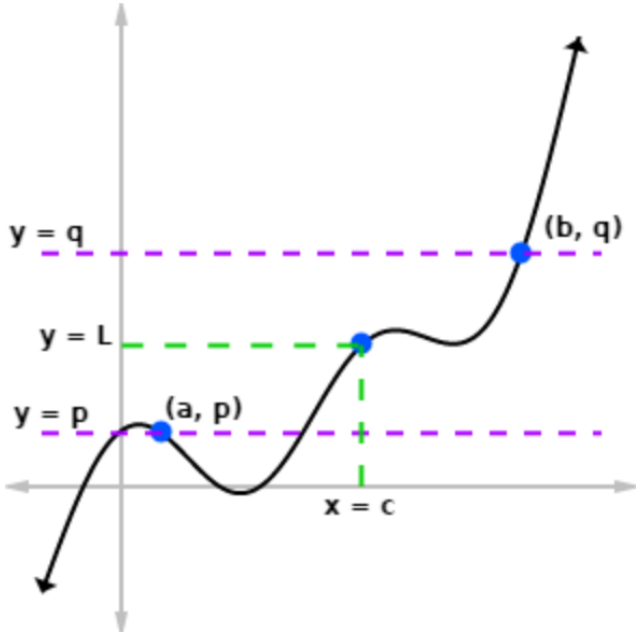
4. $g(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=1/3$

4. For which value of *a* is the following function *continuous everywhere?*

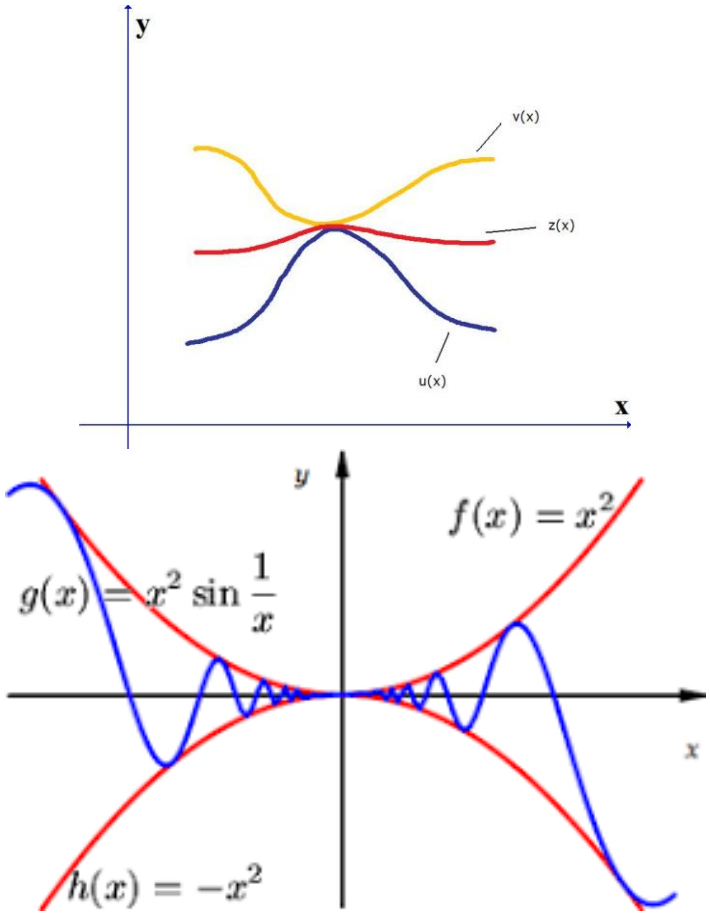
$$f(x)=\begin{cases} x^2-1 & for\ x<3 \\ 2ax & for\ x\geq 3 \end{cases}$$

5. State the *Intermediate Value Theorem.* Using the IVT, prove that the polynomial $f(x) = x^4 + 4x^3 - 20x + 11$ *must* have a root between x = 0 and x =





6. Reiew of the *Squeeze Theorem (a.k.a. Sandwich Theorem, Pinching Theorem, Two Gendarmes Theorem, Two Policemen and a Drunk Theorem).*



7. Using the Squeeze Theorem compute each of the following limits:

$$(a)\ \lim_{x\to 0} x^8 \sin^4(1/x)$$

$$(b)\ \lim_{x\to 0} x^4 \cos(1/x)$$

$$(c)\ \lim_{x\to \infty} x\sin(1/x)$$

$$(d)\ \lim_{x\to\infty} \frac{x^2\cos(2x)+\sin^3(x^{2017})}{x^3+x+5}$$

8. **Infinite limits:** Evaluate each of the following limits or explain why the limit fails to exist.

(a) $\lim_{x\to\infty} \frac{\sin x}{x}$

(b) $\lim_{x\to\infty} \frac{x^4+5x^2+2019}{(2x^2+13)^2}$

(c) $\lim_{x\to 1} \frac{x-3}{x^2+2x-4}$

(d) $\lim_{x\to 1} \frac{x-3}{x^2+2x-4}$

(e) $\lim_{h\to 4} \frac{x-4}{|4-x|}$

(f) $\lim_{x\to\infty} \frac{\sqrt{9x^2-3}}{7x^2+2x-4}$

9. Trigonometric limits: Evaluate each of the following limits or explain why the limit fails to exist.

1. $\lim_{x\to 0} \frac{\sin 4x}{x}$
2. $\lim_{x\to 0} \frac{\tan 5x}{x}$
3. $\lim_{x\to 0} \frac{\sin 2x}{\sin 8x}$
4. $\lim_{x\to\infty} \frac{\sin 13x}{x}$
5. $\lim_{x\to 0} \frac{\cos 3x}{x}$
6. $\lim_{x\to 0+} x\sin\left(\frac{1}{x}\right)$
7. $\lim_{x\to 0} \frac{\cos 11x}{\cos 13x}$
8. $\lim_{x\to 0} \frac{\tan^2 x}{x^2}$
9. $\lim_{x\to 0} \frac{\sin^2 x}{x}$
10. $\lim_{x\to 0+} \frac{|x|}{x}$
11. $\lim_{x\to 5-} \frac{x(x-5)(x-3)^2}{|x-5|}$
12. $\lim_{x\to 0} \frac{1-\cos x}{x}$
13. $\lim_{x\to 5} \sqrt{\frac{x-5}{x+1}}$
14. $\lim_{x\to 0} \frac{\sin(\sin x)}{\sin x}$
15. $\lim_{x\to 0} x \csc x$
16. $\lim_{x\to 3-} \frac{(x+4)(x-3)}{|x-3|}$
17. $\lim_{x\to 0} \cos(1/x)$
18. $\lim_{x\to 3-} \sqrt{9-x^2}$

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