MATH 161 PRACTICE TEST III (REVISED VERSION)

Covering sections 3.5, 3.6, 3.9, 4.1, 4.2, 4.3, 4.5, 4.7, 4.9, 5.1 of Stewart

1) Find an anti-derivative of each of the following functions. Show your work!

$(a)\quad (1+3x)^{2.9}$

$(b)\quad \dfrac{x}{(1+x^2)^{19}}$

$(c)\quad \dfrac{\sqrt{\arctan x}}{1+x^2}$

2) Find the *indefinite integral* of each of the following functions. Show your work!

$(a)\quad x^5\sec(x^6)\tan(x^6)$

$(b)\quad x^3e^{5+8x^4}$

$(c)\quad \dfrac{(arc\sin x)^3}{\sqrt{1-x^2}}$

3) Solve the following initial value problem:

$$\frac{dy}{dt}=t^3\cos(t^4)+t+4$$ given that y = 3 when t = 0.

4) *Verify* the following anti-differentiation formula:

$$\int x\sin(2x)\,dx=-\frac{1}{2}x\cos(2x)+\frac{1}{4}\sin(2x)+C$$

5) A cylindrical bar of radius R and length L (both in meters) is put into an oven. As the bar gains temperature, its radius decreases at a constant rate of 0.05 meters per hour and its length increases at a constant rate of 0.12 meters per hour. Fifteen minutes after the bar was put into the oven, its radius and length are 0.4 and 3 meters respectively. At what rate is the volume of the bar changing at that point? Be sure to include units.

6) Compute each of the following. *Simplify* your answers as much as possible.

$(a)\ \sum_{k=0}^{2}\frac{k}{k+1}$ $\qquad (b)\ \sum_{i=1}^{2}\left(i^4-2i\right)$ $\qquad (c)\ \sum_{j=1}^{5}\ln j$

$(d)\ \sum_{m=1}^{2018}m$ $\qquad (e)\ \sum_{j=1789}^{2017}\frac{1}{j}-\frac{1}{j+1}$ $\qquad (f)\ \sum_{k=1}^{2017}(-1)^k$

$(c)\ \sum_{j=1}^{5}\ln j$ $\qquad (d)\ \sum_{m=1}^{2018}m$ $\qquad (e)\ \sum_{j=1789}^{2017}\frac{1}{j}-\frac{1}{j+1}$

(f) $\sum_{j=1}^{1000} \ln \frac{j}{j+1}$

7) Find an anti-derivative of each of the following functions. Show your work!

(a) $\tan x$ (b) $\tan^2 x$

(b) $\sin^2 x$ *Hint:* Recall that $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

(c) $\cos^2 x$ *Hint:* Recall that $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

(d) $\sec^4 x$ *Hint:* Write $\sec^4 x = (\sec^2 x)(\sec^2 x)$ and then apply a basic identity.

8) Solve the initial value problem:

$$\frac{dy}{dx} = \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}x\right) - \frac{2\ln x}{x}$$

given that $y(1) = 13$.

9) Evaluate each of the following indefinite integrals:

(a) $\int \frac{\cos x}{\sin x + 13}\, dx =$

(b) $\int x\sin(x^2+5)\, dx =$

(c) $\int x^2(11x^3+99)^{51}\, dx =$

10) Calculate each of the following sums. Simplify each answer.

(a) $\sum_{j=3}^{4} \sin\left(j\frac{\pi}{2}\right)$ (b) $\sum_{k=2}^{4} \frac{1}{k-1}$ (c) $\sum_{m=1}^{5} m^2$

11) Evaluate each of the following. Simplify

(a) $\sum_{j=0}^{3} \frac{j}{j+3}$ ((b) $\sum_{k=0}^{4} \ln(k+1)$

(c) $\int_0^9 4\sqrt{81-x^2}\, dx$

12) Consider the following table of data:

x	-1.00	-0.25	0.50	1.25	2.00
F(x)	0.0000	2.6522	4.8755	6.8328	8.6790

Approximate the area below the graph of y = F(x) above the interval [-1, 2] using:
(a) left endpoints. *Sketch.* (b) right endpoints. *Sketch.*

13) Solve the following initial value problem:

$$\frac{dy}{dt} = \frac{2}{1+4t^2} - \frac{3}{t+1}$$

given that y(0) = 11.

14) Verify the following integration formula:

$$\int \frac{x^2}{\sqrt{2x+3}}\,dx = \frac{1}{5}\sqrt{2x+3}\,\left(6-2x+x^2\right)+C$$

15) At time *t*, in seconds, the velocity, *v*, in miles per hour, of Albertine's new Prius is given by $v(t) = 5 + 0.8t^2$ for $0 \le t \le 8$.
Use $\Delta t = 2$ to estimate the distance traveled during this time. Find the left-hand and right-hand and the average of the two. *Sketch!*

16) Octavius, the giant octopus, escaped after all. He was being kept in a temporary tank near the harbor—apparently even less secure than his tank at the zoo. He managed to get into the bay, but the coast guard could keep track of him with a homing device that had been attached to Octavius at the zoo.
(a) The coast guard station is 2 kilometers (2000 meters) down the beach from where Octavius entered the bay. If the octopus was moving directly away from the shore at a constant rate of 25 meters per minute, how fast was the distance between the coast guard and Octavius changing when the octopus was 200 meters from the shore?
(b) At what rate is the angle formed by the beach and the line that gives distance from the coast guard station to Octavius changing when Octavius is 200 meters from the shore?

17) (a) State **Rolle's Theorem**.

(b) Using Rolle's Theorem, prove that the function

$$g(x) = (x - 2)\ln(x + 1) + x\sin(4\pi x)$$

has *at least one* critical point between x = 0 and x = 2? Explain!

18) (a) State the **Mean Value Theorem**.

(b) Show how the Mean Value Theorem applies to the function

$f(x) = 4 + \ln x$ on the interval $[1, e^3]$. Sketch! Find explicitly the *c* value.

19) Explain why any two anti-derivatives of a function F(x) must differ by a constant.

Make it clear how you obtained your answer.

20) A warehouse orders and stores boxes. The cost of storing boxes is proportional to *q*, the quantity ordered. The cost of ordering boxes is proportional to 1/*q* because the warehouse gets a price cut for larger orders. The total cost of operating the warehouse is the sum of ordering costs and storage costs. What value of q gives the minimum cost?

21) For a science fair project, Albertine needs to build cylindrical cans with volume 300 cubic centimeters. The material for the side of a can costs 0.03 cents per cm^2, and the material for the bottom and top of the can costs 0.06 cents per cm^2. What is the cost of the least expensive can that she can build?

22) The equation below implicitly defines a hyperbola.

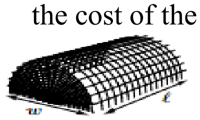
$x^2 - y^2 = 2x + xy + y + 2.$

a. Find $\frac{dy}{dx}$.

b. Consider the two points (4, 2) and (2, −1). Show that one of these points lies on the hyperbola defined above, and one does not.

c. For the point in part (b) which lies on the hyperbola, find the equation of the line that is tangent to the hyperbola at this point.

23) *[University of Michigan]* A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh midwest winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure below). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet), which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ, is $\$1 + 0.001\ell$ per square foot.



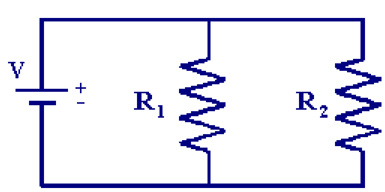
a. Write an equation for the cost of a hoophouse in terms of ℓ and w. (Hint: The surface area of a cylinder of height ℓ and radius r, not including the circles on each end, is $A = 2\pi r\ell$.)

b. Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.

24) If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an R-ohm resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 is decreasing at the rate of 1 ohm/sec and R_2 is increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ ohms and $R_2 = 50$ ohms?



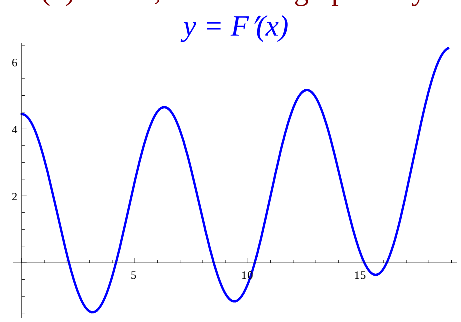
25) State the *Extreme Value Theorem*. What happens if the closed interval is replaced by an open interval? Is continuity necessary?

26) The sum of a positive number and twice its reciprocal is to be as small as possible. What is the number?

27) Find the point on the line y = 2x + 5 that is *closest* to the origin.

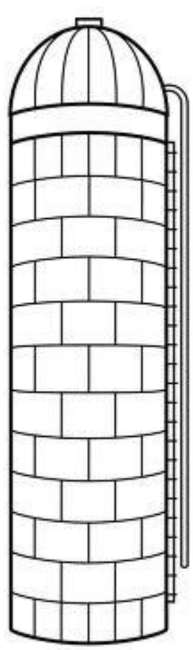
28) Find the points on the hyperbola $x^2/4 - y^2/9 = 1$ that are *closest* to the point (-1, 1).

29) Given the graph of y = F′(x) below, sketch the graphs of y = F′′(x) and y = F(x).



30) . A plasma TV screen of height 36 inches is mounted on a wall so that its lower edge is 12 inches above the eye-level of an observer. How far from the wall should the observer stand so that the viewing angle θ subtended at her eye by the TV screen is as large as possible?

31) A grain silo has the shape of a right circular cylinder surmounted by a hemisphere. If the silo is to have a volume of 504π ft^3, determine the radius and height of the silo that requires the least amount of material to build.



32) Define the function G on the interval [-1, 2] as follows:

$$G(x)=\begin{cases}13 & \text{if } -1\le x\le 0\\ 13+x^3 & \text{if } \ 0\le x\le 2\end{cases}$$

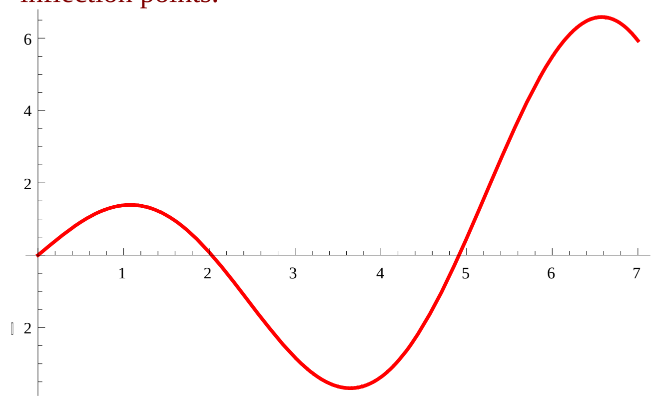
(a) Explain why G satisfies the hypotheses of the Mean Value Theorem on the interval [-1, 2]. Sketch!

(b) Determine the value of c for the function G on the interval [-1, 2] that is guaranteed by the Mean Value Theorem.

33) Below is the graph of the derivative, $F'(x)$, of a function $F(x)$.

(a) Sketch the graph of $F''(x)$.

(b) Sketch the graph of $F(x)$. Indicate local max/min, regions of increase/decrease, regions where F is concave up/down, and all inflection points.

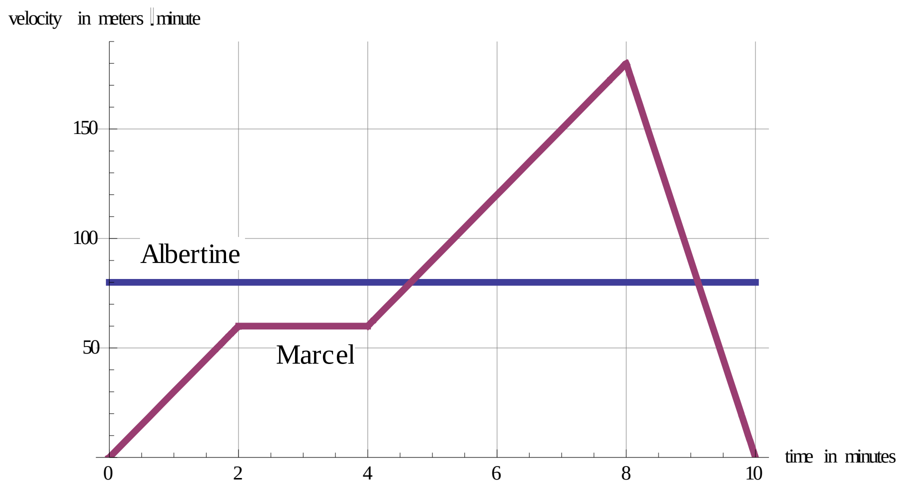


34) Verify that the function $f(x)=\arcsin x$ satisfies the hypotheses of the MVT on the interval [-1, 1]. Then find all numbers c that satisfy the conclusion of the MVT. *Sketch.*

35) (a) Use a *left-endpoint* Riemann sum with n = 4 rectangles to approximate the area between the curve $f(x) = \ln x$ and the x-axis over the interval [4, 6]. Draw a picture to illustrate what you are computing. Is this an *underestimate* or an *overestimate* of the area?

(b) Repeat (a) using right-endpoints

36) The graph below shows the ***velocities*** of two joggers, Albertine and Marcel, in meters per minute as they jog along the Champs-Élysées. Albertine and Marcel begin jogging from the same point at the same time.



(a) How far does Albertine jog in this 10 minute interval?

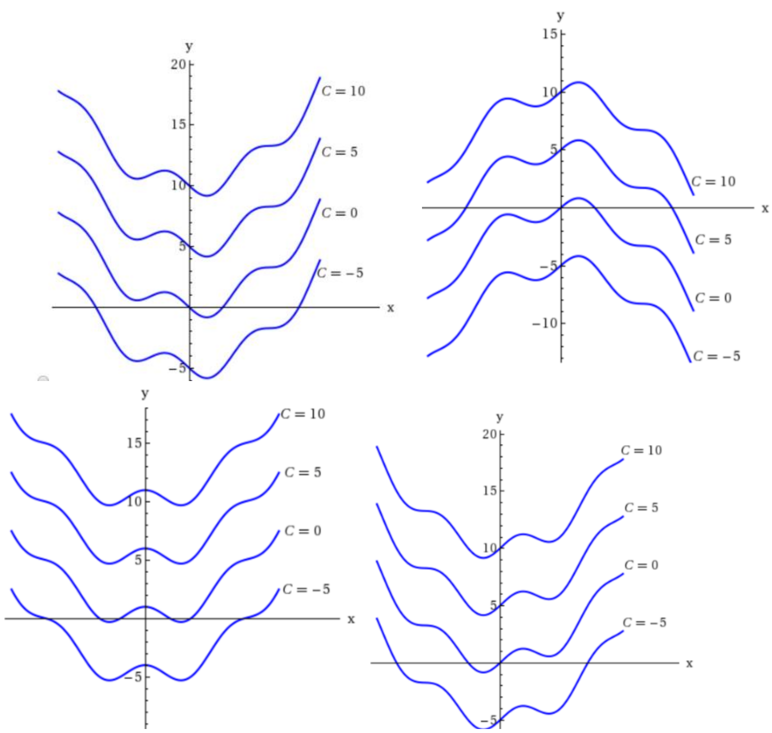
(b) How far does Marcel jog in this 10 minute interval?

(c) Who is jogging *faster* at time t = 6 minutes?

(d) Which jogger is *ahead* (i.e. has traveled the greater distance) at time t = 6 minutes? Why?

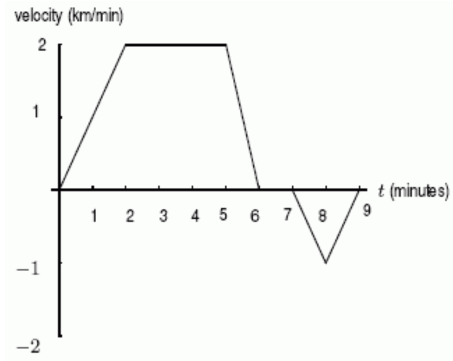
37) (Stewart) Which of the following graphs represents the set of solutions to the differential equation

$\dfrac{dy}{dx} = \cos x + \dfrac{x}{6}$? (You need not justify your answer.)

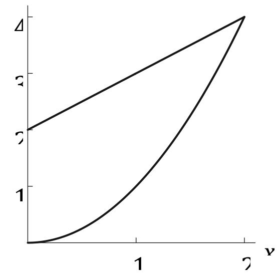


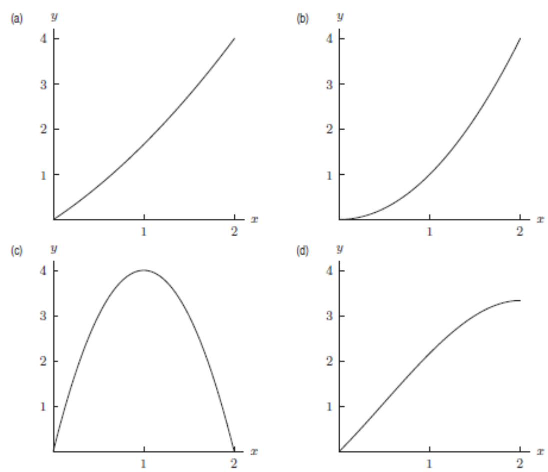
38) A car is moving along a straight road from *A* to *B*, starting from *A* at time *t* = 0. Below is the velocity (positive direction is from *A* to *B*) plotted against time.

How many kilometers away from *A* is the car at time t = 9?   *Explain!*

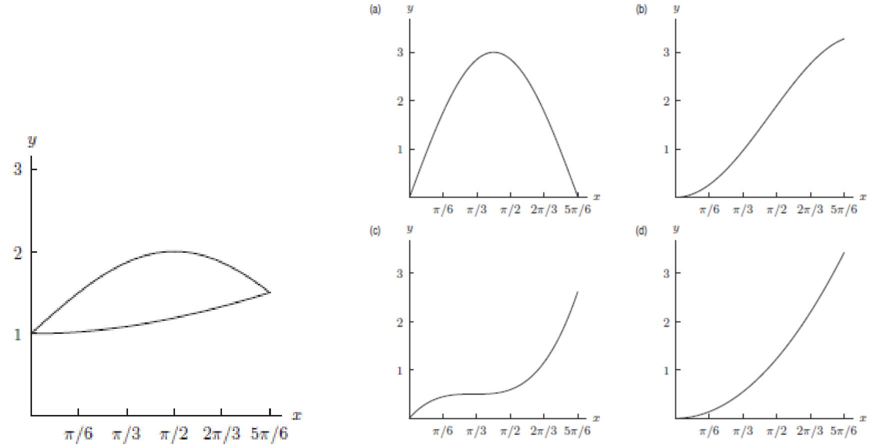


39) Consider the area between the two functions shown in the figure below. Which of the following graphs (a) through (d) represents this area as a function of x?

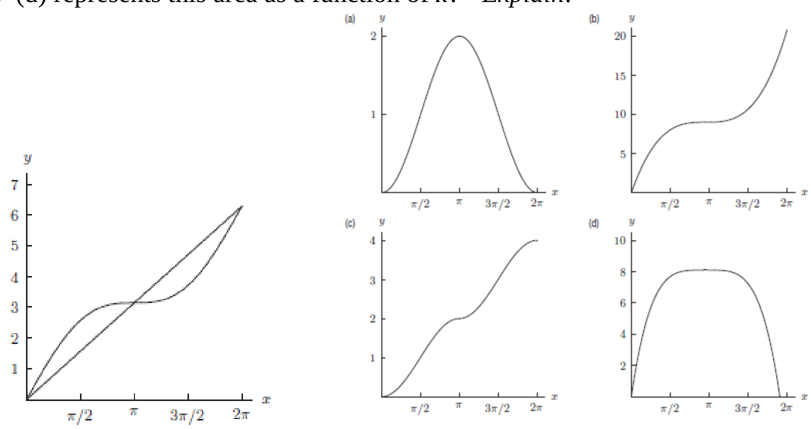




40) Consider the area between the two functions shown in the figure below. Which of the following graphs (a)–(d) represents this area as a function of x? *Explain!*



41) Consider the area between the two functions shown in the figure below. Which of the following graphs (a)–(d) represents this area as a function of x? *Explain!*



42) Given $f(x) = x^4 - 4x^3 - 8x^2 + 1$ on the interval [-5, 5].

(a) Find all critical points of *f*.

(b) Determine on which intervals *f* is increasing.

(c) Using the information obtained above, sketch the graph of y = f(x).

43) Given $H(x) = x + 2 \sin x$ on the interval $[0, 4\pi]$.

(a) Find all critical points of *H*.

(b) Determine on which intervals *H* is increasing.

(c) Sketch the graph of *H* using the above information.

44) State the **second-derivative test** for local extrema.

45) State the ***Extreme Value Theorem***. What happens if the closed interval is replaced by an open interval? Is continuity necessary?

46) Find the global extrema of $y = \cos x - 3x$ on $[0, 2\pi]$.

Sketch the graph of the function $g(x) = x^2(x-1)^2$ on the interval [0, 2]. Locate any (and all) local and global extrema.

47) (a) Let $y = (\arctan t)^7$. Compute dy/dt.

(b) Let $g(x) = \cos(\ln x)$ Compute $g'(x)$ and $g''(x)$.

(c) Let $x = (\sec(4t))^{1/2}$. Compute dx/dt.

(d) Let $z = (\ln(a+bx))^c$, where *a*, *b*, and *c* are constants. Compute dz/dx.

48) Given $f(x) = x^6 - 3x^5$ on the interval [-1, 4].

(a) Find all critical points of *f*.
(b) Determine on which intervals *f* is increasing.
(c) Find and classify all local and global extrema of *f*.
(d) On which interval(s) is *f* concave up? Find all the points of inflection.
(e) Sketch the graph of *f* using the above information.

49) Using the method of *judicious guessing*, find an *antiderivative* for each of the following functions. *Be certain to show your reasoning!*

(a) $\dfrac{(x+5)(2x-1)}{x^3}$

(b) $\dfrac{(\ln x)^{99}}{x}$

(c) $\dfrac{\sec(4x)\ \tan(4x)}{1+3\sec(4x)}$

50) A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find dx/dt when x = 3.

51) Albertine has purchased a Chevy Bolt that can accelerate from 0 ft/sec to 88 ft/sec in 5 seconds. The car's velocity is given below:

t (seconds)	0	1	2	3	4	5
v(t) (ft/second)	0	30	52	68	80	88

Using five rectangles, find upper and lower bounds (that is, over and under-estimates) for the distance traveled by Albertine's car in 5 seconds.

Learning without thought is labor lost; thought without learning is perilous.

- Confucius

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