

# MATH 161

# PRACTICE FINAL EXAM B

**PART I** (3 pts each) Answer any ten of the following eleven questions. You need not justify your answer. You may answer all eleven to obtain extra credit.

1.  $\lim_{n \rightarrow \infty} \frac{(2n^3 + n + 1)^2}{(2n^2 + 5)^3} =$

2.  $\int_{-5}^5 \sqrt{25 - x^2} \, dx =$

3.  $\frac{d}{dx} \ln(\cos x) =$

4. Let  $h(x) = \int_0^x \sin(e^{t^2}) \, dt$ . Compute  $h'(1)$ .

5.  $\frac{d^{55}}{dx^{55}} \sin x =$

6. Assume that  $a$ ,  $b$ , and  $c$  are non-zero constants. Then

$$\int_0^1 \frac{(at + b)^2}{c} \, dt =$$

7. Find an anti-derivative of

$$\frac{\cos\left(\frac{1}{x}\right)}{x^2}$$

8. Find an anti-derivative of:

$$\frac{x}{x+5}$$

9. Let  $G(x) = e^{9x} + 13$ . Find a general formula for  $G^{(k)}(0)$ , the  $k^{\text{th}}$  derivative of  $G$  evaluated at  $x = 0$ , where  $k \geq 1$ .

10. Suppose that  $f$  is defined and twice differentiable on the interval  $(0, 98)$ . If  $f'(9) = 0$  and  $f''(9) = 43$ , what, if anything, can you conclude about the point  $x = 9$ ? (For example, is it a point of inflection, a local maximum, a global minimum, etc.?)

11.  $\frac{d}{dx} x^{\ln x} =$

**PART II** (7 pts each)

Answer any 9 of the following 11 problems. You may answer more than eight for extra credit.

1. Use implicit differentiation to find  $dy/dx$ , where  $y$  is given implicitly as a function of  $x$  by the following equation:

$$x^3 + 4xy - 3y^{\frac{4}{3}} = 2x$$

2. Show that the tangents to the curve  $y = \pi \frac{\sin x}{x}$  at the points  $x = \pi$  and  $x = -\pi$ , respectively, are perpendicular.
3. The impedance  $Z$  (ohms) in a series circuit is related to the resistance  $R$  (ohms) and reactance  $X$  (ohms) by the equation  $Z = \sqrt{R^2 + X^2}$ . If  $R$  is increasing at 3 ohms/sec and  $X$  is decreasing at 2 ohms/sec, at what rate is  $Z$  changing when  $R = 10$  ohms and  $X = 20$  ohms?
4. Given the function  $f(x) = x \ln(2x) - x$  on the closed interval  $\left[\frac{1}{2e}, \frac{e}{2}\right]$ , find the global extrema and points of inflection and use this information to sketch the graph.
5. For which value or values of the constant  $k$  will the curve  $y = x^3 + kx^2 + 3x - 4$  have exactly one horizontal tangent?
6. Using an appropriate substitution, evaluate the following definite integral:

$$\int_1^4 \frac{dx}{2\sqrt{x}(1+\sqrt{x})^2}$$

7. Sketch the graph of the function  $y = \frac{(x+1)^2}{1+x^2}$ . Locate any and all local extrema and points of inflection.

8. You are designing a 1000 cm<sup>3</sup> cylindrical can whose manufacture will take waste into account. There is no waste in cutting the aluminum for the side, but the top and bottom of radius  $r$  will be cut from squares that measure  $2r$  units on a side. The total amount of aluminum used up by the can will therefore be

$$A = 8r^2 + 2\pi rh.$$

Find the ratio of  $h$  to  $r$  for the most economical can.

9. It costs Albertine  $c$  dollars to manufacture and distribute backpacks. If the backpacks sell at  $x$  dollars each, the number sold is given by

$$n = \frac{a}{x-c} + b(100-x)$$

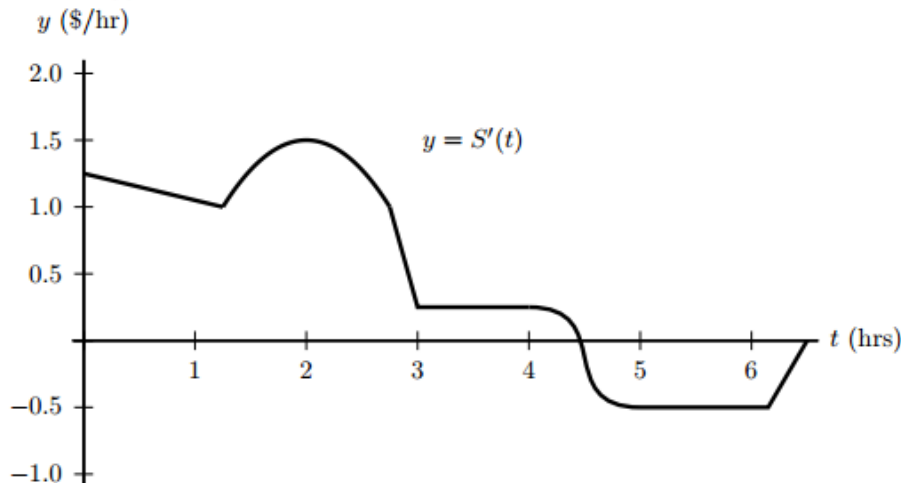
where  $a$  and  $b$  are positive constants. What will the selling price yield a maximum profit for Albertine?

10. Using the FTC, find the area bounded by the curve  $y = x^2(x-4)^2$  and the  $x$ -axis. Sketch.

11. (University of Michigan)

[8 points] Suppose that a new company named Calculus Knowledge, which provides calculus consulting work, was posted on the New York Stock Exchange over the summer. Let  $S(t)$  be a continuous and differentiable function that models the price, in dollars, of one share of Calculus Knowledge stock  $t$  hours after 9:30 am on October 6, 2014.

The graph of  $S'(t)$  for  $0 \leq t \leq 6.5$  is shown below.



Note: The graph above is the graph of  $S'(t)$ . It is **not** the graph of  $S(t)$ .

- a. [2 points] Estimate when the price of the stock is rising most quickly on October 6, 2014.

Answer: \_\_\_\_\_

- b. [2 points] According to the model  $S(t)$ , at which of the times 10 am, 11 am, 12 noon, and 1 pm was the price of one share of Calculus Knowledge stock the lowest on October 6, 2014?

Circle ONE time or circle CANNOT BE DETERMINED if the answer cannot be determined from the information provided.

10 am      11 am      12 noon      1 pm      CANNOT BE DETERMINED

- c. [2 points] On which, if any, of the following intervals does it appear that the function  $S(t)$  is always decreasing? Circle ALL correct choices or circle NONE OF THESE if appropriate.

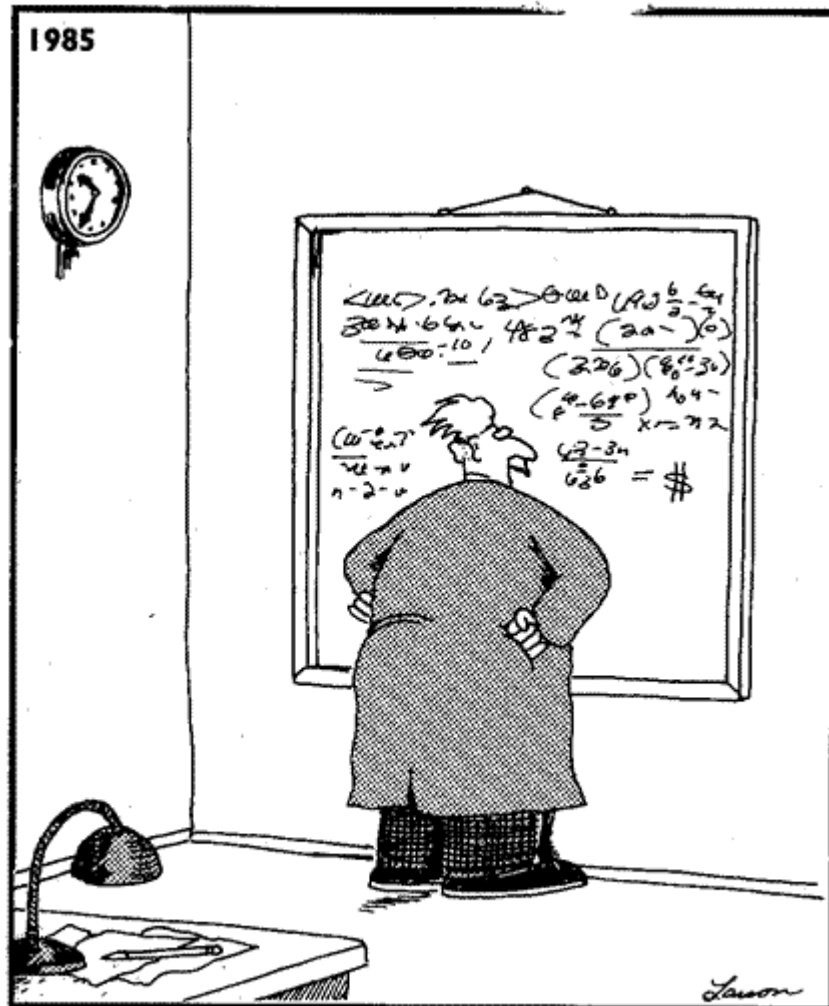
$0 < t < 1$        $2 < t < 3$        $4 < t < 5$        $5 < t < 6$       NONE OF THESE

- d. [2 points] On which, if any, of the following intervals does it appear that  $S(t)$  is linear? Circle ALL correct choices or circle NONE OF THESE if appropriate.

$0 < t < 1$        $1 < t < 2$        $3 < t < 4$        $5 < t < 6$       NONE OF THESE

*In the beginning (if there was such a thing), God created Newton's laws of motion together with the necessary masses and forces. This is all; everything beyond this follows from the development of appropriate mathematics methods by means of deduction.*

- Albert Einstein



Einstein discovers that time is actually money.