MATH 161 PRACTICE TEST II *(revised version)*

13 October 2019

Covering sections 3.1 – 3.6, 3.10; 4.1, 4.5, 4.9 of Stewart

Practice, the master of all things.

- **Augustus Octavius**

1. By using an appropriate tangent line approximation to a curve, estimate the value of $1/(0.9983)^{3/2}$. Is your answer an *underestimate* or an *overestimate*? *Sketch!*

2. *[University of Michigan]*

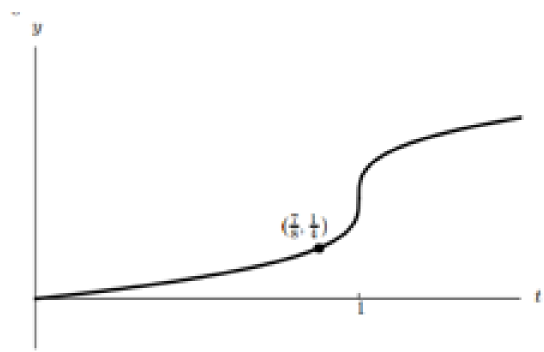
Given below is the graph of a function h(t). Suppose j(t) is the local linearization of h(t) at t = 7/ 8. Note that j(7/8) = ¼.

a. Given that h′(7 /8) = 2/3, find an expression for j(t).

b. Use your answer from (a) to approximate h(1).

c. Is the approximation from (b) an over- or under-estimate? Explain.

d. Using j(t) to estimate values of h(t), will the estimate be more accurate at t = 1 or at t = 3 /4? Explain.



3. The derivative of a continuous function g is given by

$$g'(x) = \frac{e^{-5x}(x+2)(x-3)^2(x^4+3)(x-9)}{\sin(x^3+e)+2}$$

Determine all critical points of g, and classify each as a local max, local min, or neither. Carefully explain your reasoning for each classification.

4. Using an appropriate tangent line approximation, estimate the value of each of the following quantities:

(a) $(1.00013)^{1/9}$ (b) $(0.99999)^5$ (c) $e^{0.0007}$ (d) $48.993^{1/2}$

5. Given $f(x) = x^4 - 4x^3 - 8x^2 + 1$ on the interval [-5, 5].

(a) Find all critical points of *f*.

(b) Determine on which intervals *f* is increasing.

(c) Using the information obtained above, sketch the graph of $y = f(x)$.

6. Given $f(x) = x(x - 2)^4$ on the real line.

(a) Find all critical points of *f*.

(b) Determine on which intervals *f* is increasing.

(c) Sketch the graph of *f* using the above information.

7. Given $H(x) = x + 2 \sin x$ on the interval $[0, 4\pi]$.

(a) Find all critical points of *H*.

(b) Determine on which intervals *H* is increasing.

(c) Sketch the graph of *H* using the above information.

8. Given $G(x) = x^2 / (x^2 + 3)$ on the real line.

(a) Find all critical points of *G*.

(b) Determine on which intervals *G* is increasing.

(c) Sketch the graph of *G* using the above information.

9. For each of the following functions, determine *where* the tangent line is horizontal. (The x-coordinates of these critical points are sufficient.)

$$(a)\quad y = (3x-1)^8 (2x+1)^{13}$$

$$(b)\quad y = \frac{(4x+3)^{31}}{(x-2)^{15}}$$

$$(c)\quad y = (x+1)^4 e^{3x}$$

10. State the **second-derivative test** for local extrema.

11. State the ***Extreme Value Theorem***. What happens if the closed interval is replaced by an open interval? Is continuity necessary?

12. For each of the following curves, determine all local extrema.

(a) $y = x^3/3 - x^2/2 - 2x + 1$

(b) $y = xe^{2x}$

(c) $y = x + \sin x$ on the interval $[-2\pi/3, 2\pi/3]$

(d) $y = x(1 - x)^2$

(e) $y = x^2(x^2 - 2)$

13. Sketch the curve $y = 2x + \cos x$ on the interval $[0, 6\pi]$. Find all local/global extrema.

14. For which value or values of the constant k will the curve

$$y = x^3 + kx^2 + 3x - 4$$

have *exactly one* horizontal tangent?

15. Find the global extrema of $y = \cos x - 3x$ on $[0, 2\pi]$.

16. Sketch the graph of the function $g(x) = x^2(x-1)^2$ on the interval $[0, 2]$. Locate any (and all) local and global extrema.

17. Sketch the graph of $y = e^{2/x}$. Locate any local or global extrema.

18. Suppose that the derivative of the function

$$y = f(x) \text{ is } y' = (x-1)^2(x-2).$$

Find and classify all local extrema.

19. Suppose that the derivative of the function $y = g(x)$ is

$$y' = x^2(x-2)^3(x+3).$$

Find and classify all local extrema.

20. Find the values of constants a, b, and c so that the graph of

$y = (x^2 + a) / (bx + c)$ has a local *minimum* at $x = 3$ and a local *maximum* at $(-1, -2)$.

21. (a) Let $y = (\arctan t)^7$. Compute dy/dt.

(b) Let $g(x) = \cos(\ln x)$ Compute $g'(x)$ *and* $g''(x)$.

(c) Let $x = (\sec(4t))^{1/2}$. Compute dx/dt.

(d) Let $z = (\ln(a + bx))^c$, where a, b, and c are constants. Compute dz/dx.

22. Sketch a continuous, differentiable graph with the following properties:

- local minima at 2 and 4
- global minimum at 2
- local and global maximum at 3
- no other extrema

23. Let $f(x) = x^4 - ax^2$.

(a) Find all possible critical points of f in terms of a.

(b) If $a < 0$, how many critical points does f have?

(c) If $a > 0$, find the x and y coordinates of all critical points of f.

(d) Find a value of a such that the two local minima of f occur at $x = \pm 2$.

24. Given $f(x) = x^6 - 3x^5$ on the interval $[-1, 4]$.

(a) Find all critical points of f.

(b) Determine on which intervals *f* is increasing.

(c) Find and classify all local and global extrema of *f*.

(d) On which interval(s) is *f* concave up? Find all the points of inflection.

(e) Sketch the graph of *f* using the above information.

25. Given the function $f(x) = x\ln(2x) - x$ on the closed interval $[1/(2e),\ e/2]$, find the global extrema and points of inflection and use this information to sketch the graph. Identify all local and global

26. Find equations of the tangent and normal lines to the curve

$(y - x)^2 = 2x + 4$ at the point $P = (6, 2)$.

27. *[University of Michigan]* A model for the amount of an antihistamine in the bloodstream after a patient takes a dose of the drug gives the amount, a, as a function of time, t, to be $a(t) = A(e^{-t} - e^{-kt})$. In this equation, A is a measure of the dose of antihistamine given to the patient, and k is a transfer rate between the gastrointestinal tract and the bloodstream. A and k are positive constants, and for pharmaceuticals such as antihistamine, $k > 1$.

a. Find the location $t = T_m$ of the non-zero critical point of a(t).

b. Explain why $t = T_m$ is a global maximum of a(t) by referring to the expression for a(t) or a′(t).

c. The function a(t) has a single inflection point. Find the location $t = T_I$ of this inflection point. You do not need to prove that this is an inflection point.

d. Using your expression for T_m from (a), find the rate at which T_m changes as *k* changes.

28. Let

$$y = \sqrt[3]{(x+11)(2x-1)(3x-44)(x^2+99)}\,.$$

Compute dy/dx.

29. (a) By differentiating the equation $x^2 - y^2 = 1$ implicitly, show that

$$dy/dx = x/y.$$

(b) By differentiating both sides of the equation $dy/dx = x/y$ implicitly, show that $d^2y/dx^2 = -1/y^3$.

30. Using the method of *judicious guessing*, find an *antiderivative* for each of the following functions. *Be certain to show your reasoning!*

(a) $$\frac{(x+5)(2x-1)}{x^3}$$

(b) $\dfrac{(\ln x)^{99}}{x}$

(c) $\dfrac{\sec(4x)\ \tan(4x)}{1+3\sec(4x)}$

31. Let $y = (\ln x)^x$. Find dy/dx. (*Hint:* Use logarithmic differentiation.)

32. For each of the following functions, find an *anti-derivative*. Use the method of "judicious guessing" whenever possible.

(a) 8 cos x

(b) $x^3 + 3x + 1$

(c) $\sec^2 x$

(d) sin(5x)

(e) sec(3x+1) tan(3x+1)

(f) $(3x + 11)^{1/2}$

(i) 1/(3x – 4)

33. Find equations of the tangent and normal lines to the implicitly defined curve

$2xy + \pi \sin y = 2\pi$ at the point Q = (1, π/2).

34. Show that the normal line at *any* point of the circle $x^2 + y^2 = a^2$ passes through the origin.

35. A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find dx/dt when x = 3.

36. Sketch the graph of the function $F(t) = t^{1/3}\,(t^2 - 63)$. Identify all zeroes of *F*, as well as any and all local and global extrema. Find any (and all) inflection points. What is the equation of the tangent line to

y = F(t) at t = 0?

37. Using the method of *judicious guessing*, find an anti-derivative of each of the following functions:

a) $\pi + x^3 + \dfrac{1}{\pi x^3}$

b) $\cos 4x - 3 \sin 3x + \sec^2(4x)$

c) $\dfrac{x^3 + x^2 + x + 1}{x}$

d) $(x^3 - 3)(x^2 - 4)$

e) $x^4 + \sin x + e^{1789x}$

f) $x \sin(x^2) + 1$

g) $\sec^2(13x) + (\sec x)(\tan x)$

h) $\dfrac{\arctan x}{1 + x^2}$

i) $x^4(3 + 4x^5)^6$

j) $\dfrac{e^x}{\sqrt{1 - e^{2x}}}$

k) $\dfrac{1}{x(4 + 5\ln x)}$

l) $\dfrac{\arcsin x}{\sqrt{1 - x^2}}$

38. Does $F(x) = x^3 + 2x + \tan x$ have any local extrema. Why?

39. Suppose that *a* is a positive constant. Consider the function

$$f(x) = x^3/3 - 4a^2x.$$

Determine all local and global extrema of *f* on the interval [-3a, 5a].