MATH 161 SOLUTIONS: QUIZ I 5 SEPT 2019

FYI: You will receive little or no credit if you use theorems of calculus such as **L'Hôpital's rule** that you may have learned in high school. Exceptions are made only if the student can prove the theorem.

Compute each of the following limits or show why the limit fails to exist. Circle each answer. *Justify your reasoning.*

(a) [6 pts] $\lim_{x\to 1}\left(\frac{(x-5)^4}{x^{2019}+4x-6}+\frac{6x^2+x-2}{2x-1}\right)$

Solution:

$Note\ that\ as\ x\to 1, neither\ dominator\ tends\ toward\ 0.$

$So\ we\ can\ use\ the\ limit\ law\ that\ states\ the\ limit$

$of\ a\ quotient\ is\ the\ quotient\ of\ the\ limits:$

$$\frac{(x-5)^4}{x^{2019}+4x-6}+\frac{6x^2+x-2}{2x-1}\to\frac{(1-5)^4}{1^{2019}+4(1)-6}+\frac{6(1)^2+1-2}{2(1)-1}$$

$$\to\frac{(-4)^4}{1+4-6}+\frac{6+1-2}{2-1}=-256+5=-251$$

(b) [6 pts] $\lim_{x\to 1}\frac{3x^2-7x+4}{x^4-1}$

Solution: This limit is an *indeterminate form*, 0/0, so we must look toward algebraic help. Toward this end, we factor:

As long as $x\neq 1$, $\frac{3x^2-7x+4}{x^4-1}=\frac{(3x-4)(x-1)}{(x^2+1)(x+1)(x-1)}=\frac{3x-4}{(x^2+1)(x+1)}$

Hence as $x\to 1$, $\frac{3x^2-7x+4}{x^4-1}=\frac{3x-4}{(x^2+1)(x+1)}=\frac{3(1)-4}{((1)^2+1)(1+1)}=-\frac{1}{4}$

(c) [6 pts] $\lim_{x\to -4} \frac{\sqrt{x^2+9}\ -5}{x+4}$

Solution: This limit is an *indeterminate form*, 0/0, so we must look toward algebraic help. Here we should rationalize the numerator, viz,

$$\frac{\sqrt{x^2+9}\ -\ 5}{x+4} = \frac{\left(\sqrt{x^2+9}\ -\ 5\right)}{(x+4)}\frac{\left(\sqrt{x^2+9}\ +\ 5\right)}{\left(\sqrt{x^2+9}\ +\ 5\right)} = \frac{(x^2+9)-25}{(x+4)\left(\sqrt{x^2+9}\ +\ 5\right)}$$
$$= \frac{x^2-16}{(x+4)\left(\sqrt{x^2+9}\ +\ 5\right)}$$

Note that we *do not* expand the denominator.

$= \frac{(x+4)(x-4)}{(x+4)\left(\sqrt{x^2+9}+5\right)} = \frac{x-4}{\sqrt{x^2+9}+5}$ since we assume that $x \neq -4$

Finally $as\ x \to 4$

$$\frac{x-4}{\sqrt{x^2+9}\ +\ 5} \to \frac{-8}{\sqrt{(-4)^2+9}\ +\ 5} = -\frac{8}{\sqrt{25}+5} = -\frac{4}{5}$$

(d) [6 pts] $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5}$

Solution: This limit is an *indeterminate form*, 0/0, so we must look toward algebraic help. Here we begin by combining the two fractions in the numerator:

Now $\frac{\frac{1}{x}-\frac{1}{5}}{x-5} = \frac{\frac{5}{5x}-\frac{x}{5x}}{x-5} = \frac{\frac{5-x}{5x}}{x-5} = \frac{-(x-5)}{5x(x-5)} = -\frac{1}{5x} \to -\frac{1}{25}\quad as\ x \to 5$

The limits of my language are the limits of my world.

- Ludwig Josef Johann Wittgenstein, **Tractatus Logico-Philosophicus**