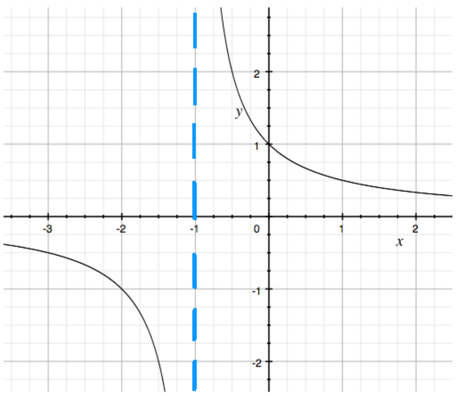
MATH 161 SOLUTIONS: QUIZ II

FRIDAY THE 13TH

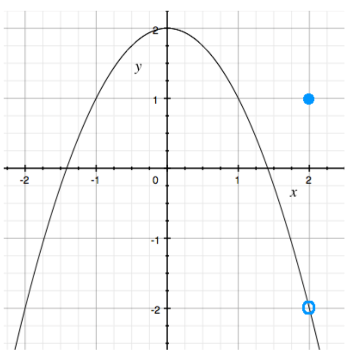
SEPTEMBER 2019

1. *[8 pts]* Give the **type of discontinuity** for each function below at the given point.

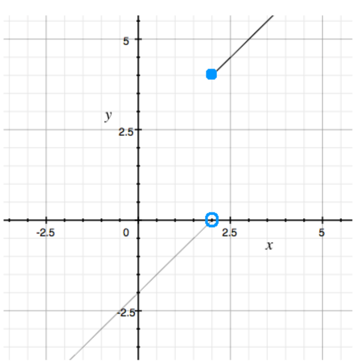
(a) At x = -1 **answer: infinite**



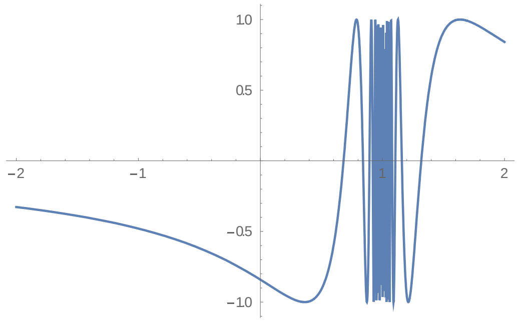
(b) At x = 2 answer: **removable**



(c) At x = 2 answer: **jump**

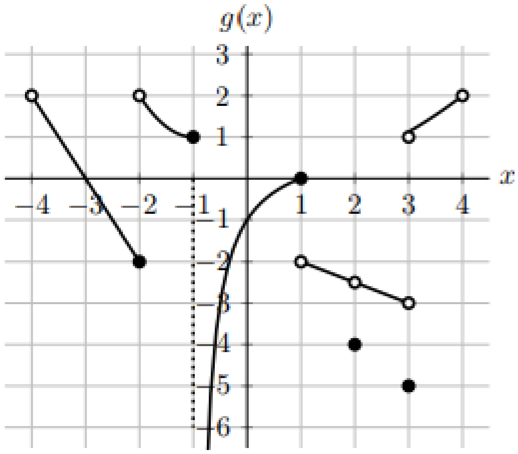


(d) At x = 1 answer: **essential**



2. *[10 pts]* Consider the functions f(x) and g(x) given by the formula and graph below.

$$f(x) = \begin{cases} 2x^3 - 2x^2 & \text{for } x \le 1, \\ x^3 + 1 & \text{for } x > 1. \end{cases}$$



(a) At which of the following values of x is the function g(x) *not* continuous? ***Circle*** the correct answer(s).

$x = -3$ $\quad$ $\boxed{x = -1}$ $\quad$ $x = 0$ $\quad$ $\boxed{x = 2}$ $\quad$ $x = 3.5$

Note that g(x) is linear on each of the intervals (−4, −2), (1, 2) and (2, 3). All your answers below should be *exac*t. *If any of the quantities do not exist, write DNE.*

(b) Find $\lim_{x\to 2}(2f(x) + g(x))$

Solution: $\lim_{x\to 2}(2f(x) + g(x)) = 2\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2(9) + (-2.5) =$ $\mathbf{15.5}$

(c) $Find \lim_{x\to\infty} \frac{f(2x)}{x^3}$

Solution: $Since\ x > 1,\ \frac{f(2x)}{x^3} = \frac{(2x)^3+1}{x^3} = \frac{8x^3+1}{x^3} \to \mathbf{8}\ as\ x \to \infty.$

(d) $Find \lim_{x\to\infty} g(x^2e^{-x}+3)$

Solution: Begin by observing that $x^2e^{-x} = \frac{x^2}{e^x} \to 0\ as\ x \to \infty$. Furthermore, $x^2e^{-x} > 0\ for\ all\ x.$

Thus $as\ x \to \infty, x^2e^{-x}+3 \to 3^+$.

And so, $\lim_{x\to\infty} g(x^2e^{-x}+3) = \lim_{x\to 3^+} g(3) = \mathbf{1}$

(e) *For which value(s) of p does* $\lim_{x\to p+} g(x) = 1$

Solution: Looking at the graph of g(x), we find that **p = -3.5**, and **p = 3** are solutions.

3. *[6 pts]* Using the IVT explain why the function f(x) = x + 3 – 2 sin x *must* have *at least one real root.*

Solution: *Note that f(0) = 3 > 0 and* $f(-\pi)$= - π + 3 – 2 *sin* $(-\pi)$ = 3 – $\pi < 0$.

Also note that f is continuous on the interval [$-\pi$*, 0] since f is a sum of continuous functions. Since f(*$-\pi$*) < 0 < f(0), the IVT guarantees the existence of a root of the equation G(x) = 0 in the interval [*$-\pi$*, 0].*

4. *[6 pts]* Compute each of the following limits. As usual, show your work.

(a) Find $\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x}$

Solution: *Since* $\lim_{x\to\pi/2} x \neq 0$ we can use the limit law $\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} = \frac{\lim_{x\to\pi/2} \sin x}{\lim_{x\to\pi/2} x} = \frac{1}{\left(\frac{\pi}{2}\right)} = \frac{2}{\pi}$

(b) Find $\lim_{x\to 0} \sqrt{\frac{\sin 9x}{x}}$

Solution: Using the limit theorems as well as continuity of the square root function:

$$\lim_{x\to 0} \sqrt{\frac{\sin 9x}{x}} = \sqrt{\lim_{x\to 0} \frac{\sin 9x}{x}} = \sqrt{\lim_{x\to 0} \frac{\sin 9x}{x}} = \sqrt{9 \lim_{x\to 0} \frac{\sin 9x}{9x}} = \sqrt{9(1)} = \mathbf{3}$$

(c) Find $\lim_{x\to 0} \frac{\tan(21x)}{\text{x}\cos(3x)}$

Solution:

$$\lim_{x\to 0} \frac{\tan(21x)}{\text{x}\cos(3x)} = \lim_{x\to 0} \frac{\sin(21x)}{\text{x}\cos(3x)\cos(21x)}$$

$$= \left(\lim_{x\to 0} \frac{1}{\cos(3x)\cos(21x)}\right)\left(\lim_{x\to 0} \frac{\sin(21x)}{x}\right) = \frac{1}{(1)(1)}(21) = 21$$

5. Consider the rational function *F* defined by $F(x) = \frac{2x^3 + x^2 - 3x}{2x^2 - 5x - 12}$

(a) *[2 pts]* Where is ***F*** *undefined*? (*Hint:* Your answer should consist of two x values.)

Solution: Begin by factoring: $\text{F(x)} = \frac{\text{x}(2\text{x}+3)(\text{x}-1)}{(2\text{x}+3)(\text{x}-4)}$

Now F is undefined when its denominator is 0:

This occurs at ***x = -3/2 and at x = 4.***

(b) *[4 pts]* At which of the two points of discontinuity does ***F*** have a removable discontinuity?
How would you define ***F*** at the removable discontinuity, x = p, to obtain a continuous function at x = p?

Solution: The removable discontinuity occurs at p = -3/2.

Since $\text{F(x)} = \frac{\text{x}(2\text{x}+3)(\text{x}-1)}{(2\text{x}+3)(\text{x}-4)}$, it follows that when $\text{x} \neq -\frac{3}{2}$,

$$\text{F(x)} = \frac{\text{x}(\text{x}-1)}{\text{x}-4} \to \frac{-\frac{3}{2}\left(-\frac{3}{2}-1\right)}{-\frac{3}{2}-4} = -\frac{15}{22} \text{ as x} \to -\frac{3}{2}.$$

So, since this limit exists, the function **has a continuous extension at x = -3/2.**

In other words, x = -3/2 is a **removable discontinuity**.

Furthermore, we should assign the value **-15/22** to F(-3/2).